MAC 2313 - Period: _____ Name: KEY

Quiz 7

April 11, 2019

Show your work to earn full credit.

1. Determine if $\vec{F} = \left\langle -\frac{y}{x^2}, \frac{z^2}{y} + \frac{1}{x}, 2z\ln y \right\rangle$ is conservative. If $\vec{F}$ is conservative, find a representative function f such that $\vec{F} = \nabla f$. (*3 points*)

$\frac{\partial P}{\partial y} = -\frac{1}{x^2} = \frac{\partial Q}{\partial x}$, $\frac{\partial P}{\partial z} = 0 = \frac{\partial R}{\partial x}$, $\frac{\partial Q}{\partial z} = \frac{2z}{y} = \frac{\partial R}{\partial y} \Rightarrow \boxed{\vec{F} \text{ conservative}}$

① So $f = \int P\,dx + g(y,z) = \frac{y}{x} + g(y,z)$

② $\frac{\partial f}{\partial z} = R \iff g_z(y,z) = 2z\ln y \Rightarrow g(y,z) = \int 2z\ln y\,dz + h(y)$

$\iff g(y,z) = z^2\ln y + h(y)$

③ $\frac{\partial f}{\partial y} = Q \iff \frac{1}{x} + \frac{z^2}{y} + h'(y) = \frac{z^2}{y} + \frac{1}{x} \Rightarrow h'(y) = 0 \Rightarrow h(y) = C.$

So letting $C=0$ we get $\boxed{f(x,y,z) = \frac{y}{x} + z^2\ln y}$

2. Using vector field $\vec{F}$ from Problem 1, evaluate $\int_C \vec{F}\cdot d\vec{r}$ for each given C. (*5 points*)

(*i*) $C : \vec{r}(t) = \langle e, e\cos t, t^2\rangle$, for $-\frac{\pi}{4} \le t \le \frac{\pi}{4}$.

Initial point: $\vec{r}(-\pi/4) = \left(e, e\cos(-\pi/4), \left(-\frac{\pi}{4}\right)^2\right) = \left(e, \frac{e\sqrt{2}}{2}, \frac{\pi^2}{16}\right)$

Terminal point: $\vec{r}(\pi/4) = \left(e, \frac{e\sqrt{2}}{2}, \frac{\pi^2}{16}\right)$

Since initial point = terminal point & $\vec{F}$ conservative we have $\boxed{\int_C \vec{F}\cdot d\vec{r} = 0.}$

(*ii*) $C : \vec{r}(t) = \langle t, t^3, 2t\rangle$, for $1 \le t \le e$.

Initial point: $\vec{r}(1) = (1,1,2)$ Terminal point: $\vec{r}(e) = (e, e^3, 2e)$.

Since $\vec{F}$ is conservative,

$$\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(e)) - f(\vec{r}(1))$$
$$= f(e, e^3, 2e) - f(1,1,2)$$
$$= \left(\frac{e^3}{e} + 4e^2\ln e^3\right) - \left(\frac{1}{1} + 4\ln(1)\right)$$
$$= (e^2 + 12e^2) - (1) = \boxed{13e^2 - 1}$$

Problem References:
1 & 2. MAC2313 L29 NYTI Problem #1 and L29 HW Problem #16.

Problem Answers:
1. $\vec{F}$ conservative with $f = \frac{y}{x} + z^2 \ln y$
2. (i) C is closed since start and endpoint $\langle e, e\frac{\sqrt{2}}{2}, \frac{\pi^2}{16}\rangle$. So $\int_C \vec{F} \cdot d\vec{r} = 0$ (ii) $\int_C \vec{F} \cdot d\vec{r} = f(e, e^3, 2e) - f(1, 1, 2) = 13e^2 - 1$.