MAC 2313 - Period: ____
Quiz 7
April 12, 2018

Name: KEY

Show your work to earn full credit.

1. Let $\vec{F} = \langle -y, x, z\rangle$ and C is parametrized by $\vec{r}(t) = \langle \cos t, \sin t, t\rangle$, for $0 \le t \le 2\pi$.

(*i*) Is $\vec{F}$ conservative? Why or why not? (*1 point*)

Observe that $\frac{\partial P}{\partial y} = -1$ and $\frac{\partial Q}{\partial x} = 1$. As $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$,

$\vec{F}$ is not conservative.

(*ii*) Evaluate $\int_C \vec{F}\cdot d\vec{r}$. (*3 points*)

$r'(t) = \langle -\sin t, \cos t, 1\rangle$. So

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle -\sin t, \cos t, t\rangle \cdot \langle -\sin t, \cos t, 1\rangle \, dt$$
$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t + t)\, dt$$
$$= \int_0^{2\pi} (1+t)\, dt = 2\pi + \tfrac{1}{2}(2\pi)^2 = \boxed{2\pi + 2\pi^2}$$

2. Let $\vec{F} = \langle 2xz + y^2, 2xy, x^2 + 15z^2\rangle$. For each given C, evaluate $\int_C \vec{F}\cdot d\vec{r}$. (*4 points*)

(*i*) $C : \vec{r}(t) = \langle t^2, t+3, 3t-1\rangle$, for $0 \le t \le 1$.

Note: $\frac{\partial P}{\partial y} = 2y = \frac{\partial Q}{\partial x}$, $\frac{\partial P}{\partial z} = 2x = \frac{\partial R}{\partial x}$, and $\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y} = 0$. So $\vec{F}$ is conservative.

Observe that $\vec{F} = \nabla f$ with $f = x^2 z + xy^2 + 5z^3$. Thus,

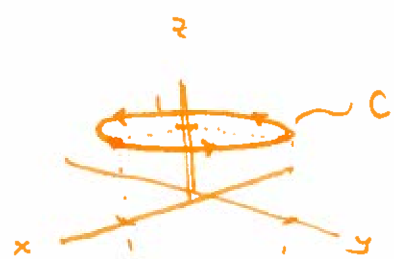
$$\int_C \vec{F}\cdot d\vec{r} = \Big[f \Big]_{\vec{r}(0)}^{\vec{r}(1)} = \Big[f \Big]_{(0,3,-1)}^{(1,4,2)} = 58 - (-5) = \boxed{63}$$

$f(1,4,2) = (1)^2\cdot(2) + (1)(4)^2 + 5(2)^3 = 58$

$f(0,3,-1) = 0 + 0 + 5(-1)^3 = -5$

(*ii*) $C : \vec{r}(t) = \langle \cos t, \sin t, 1\rangle$, for $0 \le t \le 2\pi$.

Note that C is given by



Observing this or noting that

$\vec{r}(0) = (1,0,1) = \vec{r}(2\pi)$,

we have that C is closed. As $\vec{F}$ is conservative (from (i))

we conclude that $\int_C \vec{F}\cdot d\vec{r} = \boxed{0}$.