MAC 2313 - Period: ____ Name: KEY
Quiz 8
April 18, 2019

Show your work to earn full credit.

1. Use Green's Theorem to evaluate the line integral $\int_C (e^{2x} + x^2y)\ dx + (e^{2y} - xy^2)\ dy$ where C is the circle $x^2 + y^2 = 25$ oriented clockwise. (*4 points*)

$$\oint_C (e^{2x} + x^2y)\,dx + (e^{2y} - xy^2)\,dy = -\iint_{x^2+y^2\le 25}\left[\frac{\partial}{\partial x}(e^{2y} - xy^2) - \frac{\partial}{\partial y}(e^{2x} + x^2y)\right] dA$$

Green's Thm

$$= -\iint_{x^2+y^2\le 25} (-y^2 - x^2)\, dA$$

Convert to Polar coordinates

$$= \int_0^{2\pi}\int_0^5 r^2 \cdot r\,dr\,d\theta$$

$$= 2\pi\left[\frac{1}{4}r^4\right]_0^5 = \boxed{\frac{625\pi}{2}}$$

2. Set up and evaluate a double integral for the area of the surface $x^2 + y^2 = 64$ that lies between the planes $z = 0$ and $z = 4$. (*4 points*)

Use $r(\theta, z) = \langle 8\cos\theta, 8\sin\theta, z\rangle$ with $0 \le \theta \le 2\pi$, $0 \le z \le 4$.

So $\left.\begin{array}{l} r_\theta = \langle -8\sin\theta, 8\cos\theta, 0\rangle \\ r_z = \langle 0, 0, 1\rangle \end{array}\right\} \Rightarrow r_\theta \times r_z = \langle 8\cos\theta, 8\sin\theta, 0\rangle$

Then $|r_\theta \times r_z| = \sqrt{64\cos^2\theta + 64\sin^2\theta} = 8.$

$$\text{Surface Area} = \iint_S 1\, dS = \int_0^{2\pi}\int_0^4 |r_\theta \times r_z|\, dz\, d\theta$$

$$= \int_0^{2\pi}\int_0^4 8\, dz\, d\theta$$

$$= 2\pi \cdot 4 \cdot 8$$

$$= \boxed{64\pi}$$

Note: this is surface area of side of a cylinder with $h=4$, $r=8 \Rightarrow 2\pi r h = 64\pi$.

Problem References:

1. MAC2313 L30 HW Problem #14. Answer: $\int_C \left(e^{2x} + x^2 y\right)\ dx + \left(e^{2y} - xy^2\right)\ dy = \frac{625}{2}\pi.$
2. MAC2313 L31 HW Problems #11 and #12. Answer: $\int_0^{2\pi} \int_0^4 8\ dz\ d\theta = 64\pi$