1. Use Green’s Theorem to evaluate the line integral \( \int_C (e^{2x} + x^2 y) \, dx + (e^{2y} - xy^2) \, dy \) where \( C \) is the circle \( x^2 + y^2 = 25 \) oriented clockwise. (4 points)

\[
\phi_C (e^{2x} + x^2 y) \, dx + (e^{2y} - xy^2) \, dy = - \iint_{x^2 + y^2 = 25} \left[ \frac{\partial}{\partial x} (e^{2y} - xy^2) - \frac{\partial}{\partial y} (e^{2x} + x^2 y) \right] \, dA
\]

Green’s Thm

\[
= - \int \int_{x^2 + y^2 = 25} (-y^2 - x^2) \, dA
\]

Convert to Polar Coordinates

\[
= \int_{0}^{2\pi} \int_{0}^{5} r^2 \cdot r \, dr \, d\theta
\]

\[
= 2\pi \left[ \frac{1}{4} r^4 \right]_{0}^{5} = \frac{625\pi}{2}
\]

2. Set up and evaluate a double integral for the area of the surface \( x^2 + y^2 = 64 \) that lies between the planes \( z = 0 \) and \( z = 4 \). (4 points)

Use \( r(\theta, z) = \langle 8\cos \theta, 8\sin \theta, z \rangle \) with \( 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq 4 \).

So \( r_\theta = \langle -8\sin \theta, 8\cos \theta, 0 \rangle \) \( r_z = \langle 0, 0, 1 \rangle \) \( r_\theta \times r_z = \langle 8\cos \theta, 8\sin \theta, 0 \rangle \)

Then \( |r_\theta \times r_z| = \sqrt{64\cos^2 \theta + 64\sin^2 \theta} = 8 \).

Surface Area = \( \iint_S 1 \, dS = \iint_{0}^{2\pi} \int_{0}^{4} 1 |r_\theta \times r_z| \, dz \, d\theta \)

\[
= \int_{0}^{2\pi} \int_{0}^{4} 8 \, dz \, d\theta
\]

\[
= 2\pi \cdot 4 \cdot 8 = 64\pi
\]

Note: This is surface area of side of a cylinder with \( h = 4, \quad r = 8 \) \( \Rightarrow 2\pi rh = 64\pi \).
Problem References:
1. MAC2313 L30 HW Problem #14. Answer: \( \int_C \left( e^{2x} + x^2 y \right) \, dx + \left( e^{2y} - xy^2 \right) \, dy = \frac{625}{2} \pi. \)
2. MAC2313 L31 HW Problems #11 and #12. Answer: \( \int_0^{2\pi} \int_0^4 8 \, dz \, d\theta = 64\pi \)