

Show your work to earn full credit.

1. Evaluate $\lim_{x \rightarrow 0^+} x^{\sin x}$. (2 points)

$$x^{\sin x} = e^{\sin x \ln x}$$

$$\lim_{x \rightarrow 0^+} \sin x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\csc x}$$

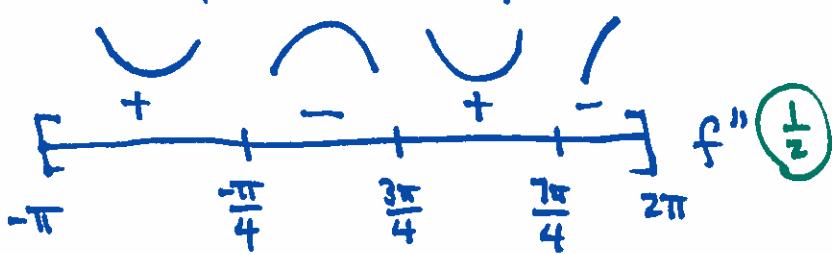
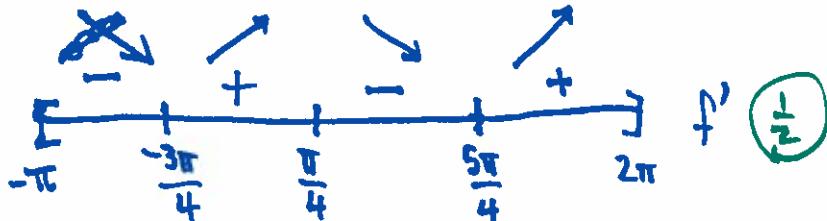
$$= \lim_{x \rightarrow 0^+} \frac{-\sin^2 x}{x \cos x} \quad \text{Apply L'H} \rightarrow \lim_{x \rightarrow 0^+} \frac{-2 \sin x \cos x}{\cos x - x \sin x} = \frac{0}{1} = 0. \quad \frac{1}{2}$$

$$\text{So, } \lim_{x \rightarrow 0^+} x^{\sin x} = e^0 = 1. \quad \frac{1}{2}$$

2. Sketch the graph of $y = \sin x + \cos x$ on the interval $[0, 2\pi]$. State intervals of increasing/decreasing, coordinates of local maxima/minima, and the x -coordinate of any inflection points. (4 points) *This is a solution that works for A or B form (up to domain)*

$$\frac{1}{2}y' = \cos x - \sin x \rightarrow y' = 0 \text{ when } \cos x = \sin x \rightarrow x = -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{1}{2}$$

$$\frac{1}{2}y'' = -\sin x - \cos x \rightarrow y'' = 0 \text{ when } \sin x = -\cos x \rightarrow x = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{1}{2}$$

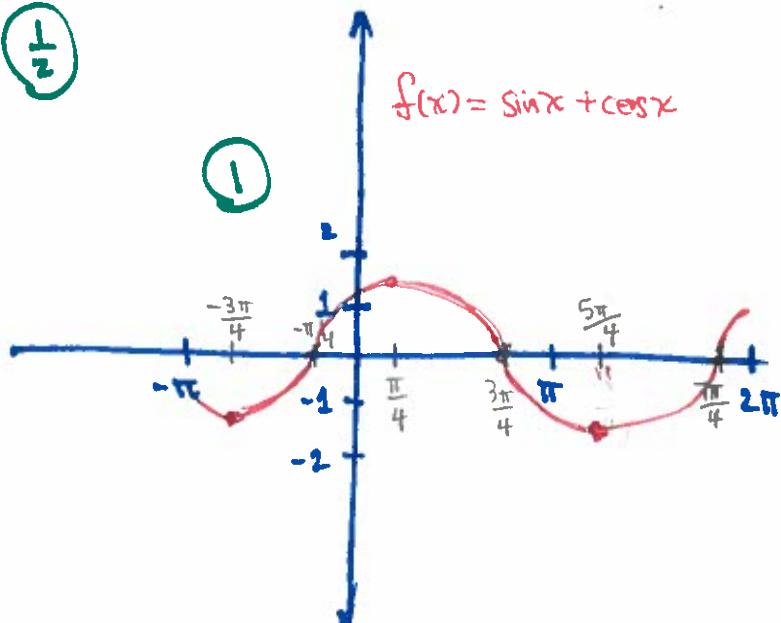


Key points:

$$f(\pi/4) = \sqrt{2}/2 + \sqrt{2}/2 = \sqrt{2}$$

$$f(5\pi/4) = \sqrt{2} = f(-\pi/4)$$

$$f(-\pi/4) = f(3\pi/4) = f(7\pi/4) = 0$$



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1. Evaluate $\lim_{x \rightarrow 0} (\cos x)^{3/x^2}$. (2 points)

$$(\cos x)^{3/x^2} = e^{\frac{3}{x^2} \ln \cos x}.$$

$$\lim_{x \rightarrow 0^+} \frac{3 \ln \cos x}{x^2} \xrightarrow[\text{L'H}]{\text{Apply L'H}} \lim_{x \rightarrow 0^+} \frac{3 \cdot \frac{-\sin x}{\cos x}}{2x} = \lim_{x \rightarrow 0^+} \frac{-3 \sin x}{2x \cos x}$$

$$\xrightarrow[\text{L'H}]{\text{Apply L'H}} \lim_{x \rightarrow 0^+} \frac{-3 \cos x}{2 \cos x - 2x \sin x} = \frac{-3}{2}.$$

So $\lim_{x \rightarrow 0} (\cos x)^{3/x^2} = e^{-3/2}$ OR $\frac{1}{e^{3/2}}$

2. Sketch the graph of $y = \cos x + \frac{1}{2}x$ on the interval $[-2\pi, 0]$. State intervals of increasing/decreasing, coordinates of local maxima/minima, and the x -coordinate of any inflection points. (4 points)

$$y = \cos x + \frac{1}{2}x$$

Same as form A.