

MAC 2311 - Period: \_\_\_\_\_  
Quiz 2 - Form A  
September 8, 2015

Name: KEY  
Please write your name and form on your paper.  
**Show your work to earn full credit.**

1. Show that  $f(x) = \frac{x-11}{2x+4}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\sin(2 \tan^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $\log_2(5-5x) - \log_2(1+x^2) = \log_2 2$ . (2 points)

MAC 2311 - Period: \_\_\_\_\_  
Quiz 2 - Form B  
September 8, 2015

Name: \_\_\_\_\_  
Write your name and form on both sheets of paper.  
**Show your work to earn full credit.**

1. Show that  $f(x) = \frac{x+11}{2x-4}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\sin(2 \cot^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $\log_6(2-x) - \log_6(7-x) = 2$ . (2 points)

MAC 2311 - Period: \_\_\_\_\_  
Quiz 2 - Form C  
September 8, 2015

Name: \_\_\_\_\_  
Write your name and form on both sheets of paper.  
**Show your work to earn full credit.**

1. Show that  $f(x) = \frac{x-5}{3x+9}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\cos(2 \tan^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $2 \log_7(x) = \log_7(2) + \log_7(x+40)$ . (2 points)

MAC 2311 - Period: \_\_\_\_\_  
Quiz 2 - Form D  
September 8, 2015

Name: \_\_\_\_\_  
Write your name and form on both sheets of paper.  
**Show your work to earn full credit.**

1. Show that  $f(x) = \frac{x+5}{3x-9}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\cos(2 \cot^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $\ln(5-5x) - \ln(1+x^2) = \ln e$ . (2 points)

**Problem References:**

1. Section 1.5 #4 and Webassign HW 2 #3.
2. Section 1.5 #39,40 and Webassign HW 2 #9.
3. Section 1.6 #28,33,34 and Webassign HW 2 #17,18.

1. Show that  $f(x) = \frac{x-11}{2x+4}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\sin(2 \tan^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $\log_2(5-5x) - \log_2(1+x^2) = \log_2 2$ . (2 points)

1. Let  $x = \frac{y-11}{2y+4} \iff 2xy+4x = y-11 \iff 4x+11 = y-2xy$   
 $\iff y(1-2x) = 4x+11$   
 $\iff f^{-1}(x) = y = \frac{4x+11}{1-2x}$   
 (1/2) for work

(1/2) Domain of  $f^{-1}(x)$ : Solve  $1-2x=0 \Rightarrow x=\frac{1}{2}$ . Domain:  $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ .

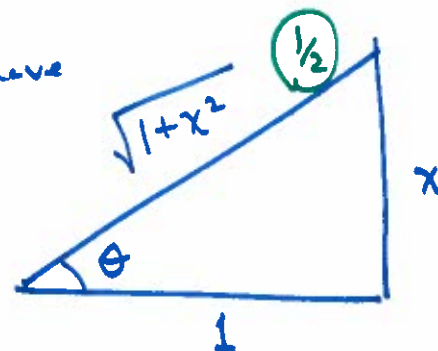
(1/2) Invertible: Use graph or let  $f(a)=f(b)$  then show  $a=b$ .

2. Let  $\theta = \tan^{-1} x$ . So  $\tan \theta = x$ . We have

Original expression is

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{x}{\sqrt{1+x^2}} \right) \left( \frac{1}{\sqrt{1+x^2}} \right) = \frac{2x}{1+x^2}$$



(1/2) Holds for  $x$  on  $(-\infty, \infty)$ .

3. Condense the expression:  $\log_2 \left( \frac{5-5x}{1+x^2} \right) = \log_2 2$

Using one-to-one property of logs:  $\frac{5-5x}{1+x^2} = 2$ .

1. Show that  $f(x) = \frac{x+11}{2x-4}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\sin(2\cot^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $\log_6(2-x) - \log_6(7-x) = 2$ . (2 points)

1. Let  $x = \frac{y+11}{2y-4} \iff 2xy - 4x = y + 11 \iff 2xy - y = 4x + 11$   
 $\iff y(2x-1) = 4x+11$   
 $\iff f^{-1}(x) = y = \frac{4x+11}{2x-1}$   
 (1/2) for work

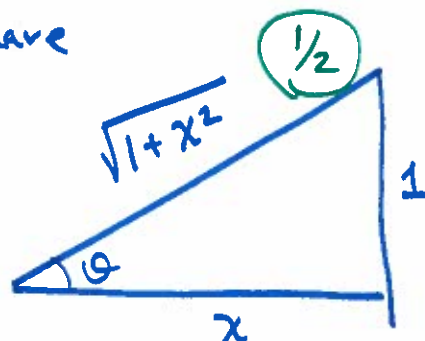
Domain of  $f^{-1}(x)$ : Solve  $2x-1=0 \rightarrow x=\frac{1}{2}$ . Domain:  $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ .  
 (1/2) Invertible: Use graph or let  $f(a)=f(b)$  then show  $a=b$ .

2. Let  $\theta = \cot^{-1} x$ . Then  $\cot \theta = x$ . We have

Original expression is

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{1}{\sqrt{1+x^2}} \right) \left( \frac{x}{\sqrt{1+x^2}} \right) = \frac{2x}{1+x^2}$$



(1/2) Holds for  $x$  on  $(-\infty, \infty)$ .

3. Condense L.H.S. of expression: (1)  $\log_6 \left( \frac{2-x}{7-x} \right) = 2$

Rewrite in exponential form: (1)  $\frac{2-x}{7-x} = 6^2 = 36$ .

## Form C

1. Let  $x = \frac{y-5}{3y+9} \iff 3xy+9x = y-5$   
 $\iff 9x+5 = y-3xy$

$\left(\frac{1}{2}\right)$  for work

$$\iff y(1-3x) = 9x+5$$

$$\iff f^{-1}(x) = y = \frac{9x+5}{1-3x} \quad \left(\frac{1}{2}\right)$$

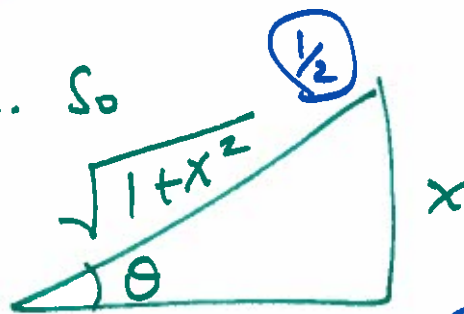
$\left(\frac{1}{2}\right)$  Domain: Solve  $1-3x=0 \Rightarrow x=\frac{1}{3}$ . D:  $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ .

$\left(\frac{1}{2}\right)$  Invertible: Use graph w/ horizontal line test  
OR

Let  $f(a)=f(b)$  and show  $a=b$ .

2. Let  $\theta = \tan^{-1}x \Rightarrow \tan\theta = x$ . So

Now the original expression  
is equivalent to



$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 - \left(\frac{x}{\sqrt{1+x^2}}\right)^2 = \frac{1-x^2}{1+x^2}.$$

$\left(\frac{1}{2}\right)$  Domain: Holds for all  $x$  in  $(-\infty, \infty)$

3. Condense R.H.S.  $\left(\frac{1}{2}\right) \log_7(x^2) = \log_7(2x+80)$

Using one-to-one  
property of logs

$\left(\frac{1}{2}\right) x^2 = 2x+80.$

1. Show that  $f(x) = \frac{x+5}{3x-9}$  is invertible, find its inverse, then find the domain of  $f^{-1}(x)$ . (2 points)
2. Rewrite  $\cos(2\cot^{-1}(x))$  as an algebraic expression of  $x$  then state the domain on which the equivalence is valid. (2 points)
3. Solve the following equation for  $x$ :  $\ln(5-5x) - \ln(1+x^2) = \ln e$ . (2 points)

1. Let  $x = \frac{y+5}{3y-9} \Leftrightarrow 3xy - 9x = y+5 \Leftrightarrow 3xy - y = 9x+5$   
 $\Leftrightarrow y(3x-1) = 9x+5$   
 $\Leftrightarrow f^{-1}(x) = y = \frac{9x+5}{3x-1}$   $\left(\frac{1}{2}\right)$

$\left(\frac{1}{2}\right)$  for work

$\left(\frac{1}{2}\right)$  Domain of  $f^{-1}(x)$ : Solve  $3x-1=0 \rightarrow x = \frac{1}{3}$ . Domain:  $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ .

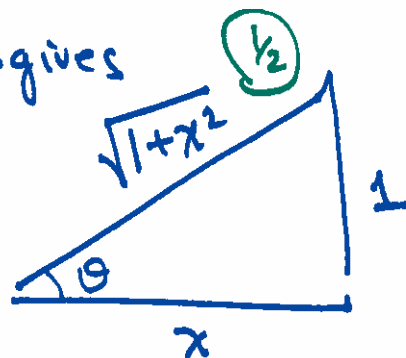
$\left(\frac{1}{2}\right)$  Invertible: Use graph or let  $f(a) = f(b)$  then show  $a=b$ .

2. Let  $\theta = \cot^{-1} x$ . So  $\cot \theta = x$ . This gives

The original expression is now

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{x}{\sqrt{1+x^2}}\right)^2 - \left(\frac{1}{\sqrt{1+x^2}}\right)^2 = \frac{x^2-1}{x^2+1}$$



$\left(\frac{1}{2}\right)$  This holds for  $x$  on  $(-\infty, \infty)$ .

3. Condense L.H.S.:  $\left(1\right) \ln\left(\frac{5-5x}{1+x^2}\right) = \ln e$

Using one-to-one property of logarithms:  $\left(1\right) \frac{5-5x}{1+x^2} = e$ .