

1. Section 2.4 # 58, 59 and Webassign HW 4 #8.
2. Section 2.6 #31, 33, 34, 37, 43 and Webassign HW 4 # 14, 16.
3. Section 2.8 #1, 5 and Webassign HW 4 #12.

MAC 2311 - Period: _____

Quiz 4 - Form A
September 22, 2015Name: KEYPlease write your name and form on your paper.
Show your work to earn full credit.

1. Find the value of the constant
- c
- that makes the function continuous. (2 points)

$$f(x) = \begin{cases} x^2 - c & : x < 4 \\ 2x + 3c & : x \geq 4 \end{cases}$$

Set $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x) \Leftrightarrow \lim_{x \rightarrow 4^-} x^2 - c = \lim_{x \rightarrow 4^+} 2x + 3c$

$$\begin{aligned} \textcircled{1} \text{ for work} &\Leftrightarrow 16 - c = 8 + 3c \\ &\Leftrightarrow 4c = 8 \\ &\Leftrightarrow c = 2 \quad \textcircled{\frac{1}{2}} \end{aligned}$$

So f is continuous at $x=4$ when $c=2$.

2. Evaluate the following limit. State any necessary theorems (2 points):
- $\lim_{x \rightarrow 0} \frac{\sin 2x \tan 5x}{x \sin 3x}$

$$\textcircled{\frac{1}{2}} \text{ Then: } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 2x \tan 5x}{x \sin 3x} &= \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \cdot \frac{\tan 5x}{5x} \cdot \frac{5x}{\sin 3x} \cdot \frac{3x}{3x} \\ \textcircled{1} \text{ for work} &= \lim_{x \rightarrow 0} 2 \left(\frac{\sin 2x}{2x} \right) \cdot 5 \left(\frac{\tan 5x}{5x} \right) \cdot \frac{1}{3} \left(\frac{3x}{\sin 3x} \right) \\ \text{Applying} &= 2 \cdot 5 \cdot \frac{1}{3} = \boxed{\frac{10}{3}} \quad \textcircled{\frac{1}{2}} \end{aligned}$$

3. ELI5 (Explain Like I'm Five): What does the Intermediate Value Theorem mean? Then use the I.V.T. to find an interval of length
- $\frac{1}{2}$
- containing a solution to
- $x^3 + 2x = -1$
- . (2 points)

$\textcircled{\frac{1}{2}} \text{ ELI5: Continuous functions don't skip values.}$

$$\text{Let } f(x) = x^3 + 2x + 1$$

$$f(0) = 1$$

$\textcircled{\frac{1}{2}} \text{ So } f \text{ continuous}$

$$f(-1) = -2$$

$\textcircled{i} \text{ By I.V.T., } (-1, 0) \text{ contains a solution. Now we need to shorten interval length. Check } f(-\frac{1}{2}) = -\frac{1}{8} < 0$

$\Rightarrow \text{Solution on } (-\frac{1}{2}, 0) \text{ } \textcircled{\frac{1}{2}}$

MAC 2311 - Period: _____

Quiz 4 - Form B

September 22, 2015

Name: KEY

Write your name and form on both sheets of paper.

Show your work to earn full credit.

1. Find the value of the constant c that makes the function continuous. (2 points)

$$f(x) = \begin{cases} x^2 - c & : x < 6 \\ 2x + 2c & : x \geq 6 \end{cases}$$

Set $\lim_{x \rightarrow 6^-} f(x) = \textcircled{1/2} \quad \lim_{x \rightarrow 6^+} f(x) \Leftrightarrow \lim_{x \rightarrow 6^-} x^2 - c = \lim_{x \rightarrow 6^+} 2x + 2c$

$\textcircled{1}$ pt for work $\Leftrightarrow 36 - c = 12 + 2c$

$$\Leftrightarrow 3c = 24 \Leftrightarrow c = \underline{\underline{8}}. \textcircled{1}$$

$\therefore \lim_{x \rightarrow 6} f(x)$ exists and $\lim_{x \rightarrow 6} f(x) = 28 = f(6)$.

$\therefore f$ is continuous at $x=6$.

2. Evaluate the following limit. State any necessary theorems (2 points): $\lim_{x \rightarrow 0} \frac{\sin 3x \sin 7x}{x \tan 2x}$

$\textcircled{1/2}$ Then $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$.

$$\lim_{x \rightarrow 0} \frac{\sin 3x \sin 7x}{x \tan 2x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \frac{\sin 7x}{x} \cdot \frac{x}{\tan 2x}$$

$\textcircled{1}$ for work $= \lim_{x \rightarrow 0} 3\left(\frac{\sin 3x}{3x}\right) \cdot 7\left(\frac{\sin 7x}{7x}\right) \cdot \frac{1}{2} \cdot \left(\frac{2x}{\tan 2x}\right)$

Applying $\textcircled{1}$ $= 3 \cdot 7 \cdot \frac{1}{2} = \boxed{\frac{21}{2}} \textcircled{1}$

3. ELI5 (Explain Like I'm Five): What does the Intermediate Value Theorem mean? Then use the I.V.T. to find an interval of length $\frac{1}{2}$ containing a solution to $x^3 - 8x = 1$. (2 points)

$\textcircled{1/2}$ ELI5: If a function is continuous on a closed interval then it doesn't skip values.

Let $f(x) = x^3 - 8x - 1$

$$f(0) = 0 - 0 - 1 = -1 \quad f(1) = 1 - 8 - 1 = -8$$

$\textcircled{1/2}$ So f is continuous

$$f(-1) = 6$$

$\textcircled{1/2}$ So $(-1, 0)$ contains a solution by I.V.T. Need to shorten interval length

(check $f(-\frac{1}{2}) = -\frac{1}{8} + 4 - 1 > 0 \Rightarrow$ Solution on $(-\frac{1}{2}, 0)$) $\textcircled{1/2}$

MAC 2311 - Period: _____

Quiz 4 - Form C

September 22, 2015

Name: KEY

Write your name and form on both sheets of paper.

Show your work to earn full credit.

1. Find the value of the constant c that makes the function continuous. (2 points)

$$f(x) = \begin{cases} x^2 - c & : x < 8 \\ 2x + 3c & : x \geq 8 \end{cases}$$

Set $\lim_{x \rightarrow 8^-} f(x) = \lim_{x \rightarrow 8^+} f(x) \Leftrightarrow \lim_{x \rightarrow 8^+} x^2 - c = \lim_{x \rightarrow 8^+} 2x + 3c$

$\textcircled{1} \text{ pt for work} \Leftrightarrow 64 - c = 16 + 3c \Leftrightarrow 48 \Leftrightarrow c = 12. \text{ } \textcircled{\frac{1}{2}}$

Thus, $\lim_{x \rightarrow 8} f(x)$ exists and $\lim_{x \rightarrow 8} f(x) = 52 = f(8)$.

Thus, f is continuous at $x = 8$.

2. Evaluate the following limit. State any necessary theorems (2 points): $\lim_{x \rightarrow 0} \frac{\sin 5x \tan 6x}{x \sin 2x}$

$\textcircled{\frac{1}{2}} \text{ Thm: } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \rightarrow \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 5x \tan 6x}{x \sin 2x} &= \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \cdot \frac{\tan 6x}{x} \cdot \frac{x}{\sin 2x} \\ &= \lim_{x \rightarrow 0} 5 \left(\frac{\sin 5x}{5x} \right) \cdot 6 \left(\frac{\tan 6x}{6x} \right) \cdot \frac{1}{2} \left(\frac{2x}{\sin 2x} \right) \end{aligned}$$

Applying the thm: $= 5 \cdot 6 \cdot \frac{1}{2} = \boxed{15} \text{ } \textcircled{\frac{1}{2}}$

3. ELI5 (Explain Like I'm Five): What does the Intermediate Value Theorem mean? Then use the I.V.T. to find an interval of length $\frac{1}{2}$ containing a solution to $x^3 - 8x = 1$. (2 points)

$\textcircled{\frac{1}{2}} \text{ ELI5: If } f \text{ is continuous on a closed interval then it doesn't skip any values. (may be alright)}$

Let $f(x) = x^3 - 8x - 1 \quad f(0) = -1 \quad f(1) = 1 - 8 - 1 = -8$

$\textcircled{\frac{1}{2}} \text{ So } f \text{ is continuous.} \quad f(-1) = -1 + 8 - 1 = 6$

$\textcircled{\frac{1}{2}} \text{ So } (-1, 0) \text{ contains a solution by I.V.T. Need to shorten interval length.}$

Check $f(-\frac{1}{2}) = -\frac{1}{8} + 4 - 1 > 0 \Rightarrow \text{solution on } (-\frac{1}{2}, 0) \text{ } \textcircled{\frac{1}{2}}$

1. Find the value of the constant c that makes the function continuous. (2 points)

$$(1/2) \quad f(x) = \begin{cases} x^2 - c & : x < 9 \\ 3x + 6c & : x \geq 9 \end{cases}$$

$$\text{Set } \lim_{x \rightarrow 9^-} f(x) = \lim_{x \rightarrow 9^+} f(x) \Leftrightarrow \lim_{x \rightarrow 9^-} x^2 - c = \lim_{x \rightarrow 9^+} 3x + 6c$$

$$(1) \text{ for work} \Leftrightarrow 81 - c = 27 + 6c$$

$$\Leftrightarrow 7c = 54 \rightarrow c = \frac{54}{7} (1/2)$$

$$\text{So } \lim_{x \rightarrow 9} f(x) \text{ exists and } \lim_{x \rightarrow 9} f(x) = 81 - \frac{54}{7} = f(9).$$

Thus, f is continuous at $x=9$.

2. Evaluate the following limit. State any necessary theorems (2 points): $\lim_{x \rightarrow 0} \frac{\sin 5x \sin 3x}{x \tan 6x}$

$$(1/2) \text{ Then: } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\text{we have } \lim_{x \rightarrow 0} \frac{\sin 5x \sin 3x}{x \tan 6x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \cdot \frac{\sin 3x}{x} \cdot \frac{x}{\tan 6x}$$

$$(1) \text{ for work} = \lim_{x \rightarrow 0} 5 \left(\frac{\sin 5x}{5x} \right) \cdot 3 \left(\frac{\sin 3x}{3x} \right) \cdot \frac{1}{6} \left(\frac{6x}{\tan 6x} \right)$$

$$\text{Applying} \quad = 5 \cdot 3 \cdot \frac{1}{6} = \boxed{\frac{5}{2}} (1/2)$$

3. ELI5 (Explain Like I'm Five): What does the Intermediate Value Theorem mean? Then use the I.V.T. to find an interval of length $\frac{1}{2}$ containing a solution to $x^3 + 2x = -1$. (2 points)

(1/2) ELI5: Continuous functions ~~between~~ don't skip values.

$$\text{let } f(x) = x^3 + 2x + 1 = 0 \quad f(0) = 1 \quad f(-1) = -1 - 2 + 1 = -2$$

(1/2) So f is continuous.

(1/2) By I.V.T., $(-1, 0)$ contains a solution. Now we need to shorten interval length. Check $f(-\frac{1}{2}) = -\frac{1}{8} - 1 + 1 = -\frac{1}{8} < 0$.

\Rightarrow solution on $(-\frac{1}{2}, 0)$ (1/2)