

MAC 2311 - Period: _____
Quiz 6 - Form A
October 13, 2015

Name: Key

Show your work to earn full credit.

1. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(2) = -1$, $f'(2) = 5$, $g(2) = -2$, and $g'(2) = 4$. Given that $h(x) = \frac{x^2}{f(x)g(x) + 1}$, find $h'(2)$. (2 points)

Let $u = x^2$ $\frac{1}{2}$ $v = f(x)g(x) + 1$ $\frac{1}{2}$
 $u' = 2x$ $v' = f'(x)g(x) + f(x)g'(x)$

So $u(2) = 4$, $u'(2) = 4$, $v(2) = 3$, $v'(2) = -14$.

$$h'(x) = \frac{uv - v'u}{v^2} \rightarrow h'(2) = \frac{(4)(3) - (4)(-14)}{(3)^2} = \frac{\frac{68}{9}}{\frac{1}{2}}$$

2. Find $f'(x)$ and $f''(x)$ given that $f(x) = \frac{e^x}{3x^2}$. (2 points)

$$f'(x) = \frac{3x^2 e^x - 6x e^x}{(3x^2)^2} = \boxed{\frac{e^x}{3x^2} \left(1 - \frac{2}{x}\right)} = f(x) \left(1 - \frac{2}{x}\right)$$

So $f''(x) = f'(x) \left(1 - \frac{2}{x}\right) + f(x) \left(\frac{2}{x^2}\right) = \boxed{\frac{e^x}{3x^2} \left(1 - \frac{4}{x} + \frac{6}{x^2}\right)}$

① for work

3. Given that $g(x) = 3 \sin x - \cos x + x^9$ determine $g^{(8)}$ and $g^{(127)}$. (2 points)

Let $f(x) = 3 \sin x - \cos x$

so $f^{(4n)} = 3 \sin x - \cos x$

$$f^{(4n+1)} = 3 \cos x + \sin x$$

$$f^{(4n+2)} = -3 \sin x + \cos x$$

$$f^{(4n+3)} = -3 \cos x - \sin x$$

① for work

so $g^{(8)} = f^{(8)} + 9!x$

$$= \boxed{3 \sin x - \cos x + 9!x}$$

Since

$$\begin{array}{r} 0 \\ 4 \sqrt{127} \\ \underline{-12} \\ \hline 07 \\ \underline{-4} \\ \hline 3 \end{array}$$

$$g^{(127)} = f^{(4(31)+3)} = \boxed{-3 \cos x - \sin x}$$

MAC 2311 - Period: _____
Quiz 6 - Form B
October 13, 2015

Name: Key _____
Show your work to earn full credit.

1. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(2) = -1$, $f'(2) = 5$, $g(2) = -2$, and $g'(2) = 4$. Given that $h(x) = \frac{2+x^2}{f(x)g(x)-3}$ find $h'(2)$. (2 points)

Let $u = 2+x^2$ $\frac{1}{2}$ $v = f(x)g(x)-3$
 $u' = 2x$ $\frac{1}{2}$ $v' = f'(x)g(x) + f(x)g'(x)$ $\frac{1}{2}$

So $u(2) = 6$, $u'(2) = 4$, $v(2) = -1$, $v'(2) = -14$.

$$h'(x) = \frac{u'v - v'u}{v^2} \frac{1}{2} \rightarrow h'(2) = \frac{(4)(-1) - (-14)(6)}{(-1)^2} = \boxed{80} \frac{1}{2}$$

2. Find $f'(x)$ and $f''(x)$ given that $f(x) = \frac{e^x}{2x^3}$. (2 points)

$$f'(x) = \frac{e^x(2x^3) - 6x^2e^x}{(2x^3)^2} = \frac{e^x}{2x^3} - \frac{3e^x}{2x^4}$$

① for work-

$$= \boxed{\frac{e^x}{2x^3} \left(1 - \frac{3}{x}\right)} \frac{1}{2}$$

$$f''(x) = f'(x) \left(1 - \frac{3}{x}\right) + f(x) \left(\frac{3}{x^2}\right) = \boxed{\frac{e^x}{2x^3} \left(1 - \frac{6}{x} + \frac{12}{x^2}\right)} \frac{1}{2}$$

3. Given that $g(x) = \sin x - 2\cos x + x^8$ determine $g^{(7)}$ and $g^{(106)}$. (2 points)

Let $f(x) = \sin x - 2\cos x$. So $g^{(7)} = f^{(7)} + 8!x$

So $f^{(4n)} = \sin x - 2\cos x$

$f^{(4n+1)} = \cos x + 2\sin x$

$f^{(4n+2)} = -\sin x + 2\cos x$

$f^{(4n+3)} = -\cos x - 2\sin x$

① for work

Since $4 \sqrt[4]{106} \approx 2.6$
 $= \frac{4}{2.6}$
 $= \frac{2}{1.3}$
 $= \frac{20}{13}$

So $g^{(106)} = f^{(4(26)+2)}$

$$= \boxed{-\sin x + 2\cos x.} \frac{1}{2}$$

Show your work to earn full credit.

1. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(2) = -1$, $f'(2) = 5$, $g(2) = -2$, and $g'(2) = 4$. Given that $h(x) = \frac{x^2 - 1}{f(x)g(x) + 2}$ find $h'(2)$. (2 points)

$$\text{Let } u = x^2 - 1 \quad v = f(x)g(x) + 2 \\ u' = 2x \quad \frac{1}{2} \quad v' = f'(x)g(x) + f(x)g'(x) \quad \frac{1}{2}$$

$$\text{So } u(2) = 3, \quad u'(2) = 4, \quad v(2) = 4, \quad v'(2) = -14.$$

$$h'(x) = \frac{uv - v'u}{v^2} \quad h'(2) = \frac{(4)(4) - (-14)(3)}{(4)^2} = \boxed{\frac{58}{16}} \quad \frac{1}{2} \\ \text{or } \boxed{\frac{29}{8}}$$

2. Find $f'(x)$ and $f''(x)$ given that $f(x) = \frac{e^x}{4x^2}$. (2 points)

$$f' = \frac{4x^2 e^x - 8x e^x}{(4x^2)^2} = \boxed{\frac{e^x}{4x^2} \left(1 - \frac{2}{x} \right)} \quad \frac{1}{2}$$

$$f'' = f'(x) \left(1 - \frac{2}{x} \right) + f(x) \left(\frac{2}{x^2} \right) = \boxed{\frac{e^x}{4x^2} \left(1 - \frac{4}{x} + \frac{6}{x^2} \right)} \quad \frac{1}{2}$$

① for work.

3. Given that $g(x) = \sin x - 3 \cos x + x^8$ determine $g^{(7)}$ and $g^{(126)}$. (2 points)

$$\text{Let } f(x) = \sin x - 3 \cos x.$$

$$\text{so } g^{(7)} = f^{(7)} + 8!x$$

$$\text{So } f^{(4n+1)} = \cos x + 3 \sin x$$

$$= \boxed{-\cos x - 3 \sin x + 8!x} \quad \frac{1}{2}$$

$$f^{(4n+2)} = -\sin x + 3 \cos x$$

$$\begin{array}{r} 031 \\ 4 \sqrt{126} \\ \underline{-124} \\ 06 \end{array} \quad \text{r.2}$$

$$f^{(4n+3)} = -\cos x - 3 \sin x$$

$$f^{(4n)} = \sin x - 3 \cos x$$

$$\text{so } g^{(126)} = f^{(4(31)+2)}$$

$$= \boxed{-\sin x + 3 \cos x} \quad \frac{1}{2}$$

① for work.

Show your work to earn full credit.

1. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(2) = -1$, $f'(2) = 5$, $g(2) = -2$, and $g'(2) = 4$. Given that $h(x) = \frac{3x^2}{f(x)g(x) - 1}$ find $h'(2)$. (2 points)

$$\text{Let } u = 3x^2 \quad v = f(x)g(x) - 1 \\ u' = 6x \quad v' = f'(x)g(x) + f(x)g'(x)$$

$$\text{So } u(2) = 12, \quad u'(2) = 12, \quad v(2) = 1, \quad v'(2) = -14$$

$$h'(x) = \frac{u'v - v'u}{v^2} \rightarrow h'(2) = \frac{(12)(1) - (-14)(12)}{(1)^2} = 180$$

2. Find $f'(x)$ and $f''(x)$ given that $f(x) = \frac{e^x}{3x^3}$. (2 points)

$$f'(x) = \frac{3x^3 e^x - 9x^2 e^x}{(3x^3)^2} = \frac{e^x}{3x^3} \left(1 - \frac{3}{x}\right)$$

$$f''(x) = f'(x) \left(1 - \frac{3}{x}\right) + f(x) \left(\frac{3}{x^2}\right) = \frac{e^x}{3x^3} \left(1 - \frac{6}{x} + \frac{12}{x^2}\right)$$

① for work

3. Given that $g(x) = 2\sin x - \cos x + x^9$ determine $g^{(8)}$ and $g^{(105)}$. (2 points)

$$\text{Let } f(x) = 2\sin x - \cos x.$$

$$\text{So } f^{(4n)} = 2\sin x - \cos x$$

$$f^{(4n+1)} = 2\cos x + \sin x$$

$$f^{(4n+2)} = -2\sin x + \cos x$$

$$f^{(4n+3)} = -2\cos x - \sin x.$$

① for work

$$\text{So } g^{(8)} = f^{(8)} + 9!x$$

$$= 2\sin x - \cos x + 9!x$$

$$\text{Since } 4 \sqrt[4]{105} \approx 1.2$$

$$\text{So } g^{(105)} = f^{(4(26)+1)}$$

$$= 2\cos x + \sin x$$

Problem References:

1. Section 3.3 #40 - 42 and Webassign HW 6 #15, 16.
2. Section 3.5 #16.
3. Section 3.6 #43 and Webassign HW 6 #7.