

Quiz 7 - Problem References

MAC 2311

1. Section 3.9 #41, 44, 45, 46.
2. Section 3.10 #23, 24, 25, 26.
3. Section 3.8 #31 and Webassign HW 7 #29, 30, 36.

Show your work to earn full credit.

1. Find the derivative of $f(x) = x^{8x} \cdot \sqrt[3]{x^4 + 2x}$. (2 points)

Take natural log of both sides:

$$\ln f = \ln[x^{8x} \cdot (x^4 + 2x)^{\frac{1}{3}}] = 8x \ln x + \frac{1}{3} \ln(x^4 + 2x) \quad (\frac{1}{2})$$

$$\text{So } \frac{1}{f} \cdot f' = 8 \ln x + 8x(\frac{1}{x}) + \frac{1}{3} \frac{4x^3 + 2}{x^4 + 2x} \quad (\frac{1}{2})$$

$$\Rightarrow f'(x) = x^{8x} \sqrt[3]{x^4 + 2x} \left(8 \ln x + 8 + \frac{4x^3 + 2}{3(x^4 + 2x)} \right).$$

2. Given $e^{3xy} = \sin(y^2)$ calculate the derivative with respect to x . (2 points)

Take derivative of both sides with respect to x .

$$(3y + 3xy') e^{3xy} = 2yy' \cdot \cos(y^2) \quad (\frac{1}{2})$$

Now solve for y' : $\frac{1}{2}$ for work

$$3xy' e^{3xy} - 2yy' \cdot \cos(y^2) = -3ye^{3xy}$$

$$\Leftrightarrow y' = \frac{-3ye^{3xy}}{3x e^{3xy} - 2y \cos(y^2)} \quad (\frac{1}{2})$$

3. Determine the value(s) of t for which $f(t) = 2 \cdot \sqrt{1-t^4} + \sin^{-1}(t^2)$ has a horizontal tangent line. (2 points)

Solve $f'(t) = 0$ for t : $\frac{1}{2}$

$$f'(t) = \frac{(\frac{1}{2})2(-4t^3)}{\sqrt{1-t^4}} + \frac{2t}{\sqrt{1-t^4}} = 0$$

$$\Leftrightarrow \frac{-2t(8t^2 - 1)}{\sqrt{1-t^4}} = 0 \rightarrow t = 0, \pm \frac{1}{\sqrt{2}}$$

1. Find the derivative of $f(x) = x^{6x^2} \cdot \cos^3(x)$. (2 points)

Take natural log of both sides.

$$\ln f = \ln [x^{6x^2} \cdot \cos^3 x] = 6x^2 \ln x + 3 \ln \cos x. \quad (\frac{1}{2})$$

$$\text{So } \frac{1}{f} \cdot f' = \frac{12x \ln x + 6x^2(\frac{1}{x})}{\cos x} + 3 \frac{-\sin x}{\cos x}$$

$$\Rightarrow f'(x) = x^{6x^2} \cos^3 x \left(12x \ln x + 6x - \frac{3 \sin x}{\cos x} \right)$$

2. Given $e^{2xy} = \cos(y^2)$ calculate the derivative with respect to x . (2 points)

$$\frac{d}{dx} e^{2xy} = \frac{d}{dx} \cos(y^2)$$

$$\Leftrightarrow (2y + 2xy')e^{2xy} = -2yy' \sin(y^2) \quad (\frac{1}{2}) \text{ for work}$$

$$\Leftrightarrow 2ye^{2xy} = -2xy'e^{2xy} - 2yy' \sin(y^2)$$

$$\text{So, } y' = \frac{-2ye^{2xy}}{2xe^{2xy} + 2y \sin(y^2)} \quad (\frac{1}{2})$$

3. Determine the value(s) of t for which $f(t) = 2 \cdot \sqrt{1-t^4} - \cos^{-1}(t^2)$ has a horizontal tangent line. (2 points)

Solve $f'(t) = 0$ for t : $(\frac{1}{2})$

$$f'(t) = \frac{\frac{1}{2} \cdot 2(-4t^3)}{\sqrt{1-t^4}} + \frac{2t}{\sqrt{1-t^4}} = 0$$

$$\Leftrightarrow \frac{-2t(2t^2 - 1)}{\sqrt{1-t^4}} = 0 \rightarrow t=0, \pm \frac{1}{\sqrt{2}} \quad (\frac{1}{2})$$

1. Find the derivative of $f(x) = x^{x^3} \cdot \sqrt[3]{x^2 + 4}$. (2 points)

$$\ln f = \ln [x^{x^3} \cdot (x^2 + 4)^{\frac{1}{2}}] = x^3 \ln x + \frac{1}{2} \ln(x^2 + 4)^{\frac{1}{2}}$$

$$\rightarrow \frac{1}{f} \cdot f' = \frac{1}{3x^2 \ln x + x^3(\frac{1}{x})} + \frac{1}{2} \frac{2x}{x^2 + 4}$$

So $f'(x) = x^{x^3} \sqrt[3]{x^2 + 4} \left(3x^2 \ln x + x^2 + \frac{x}{x^2 + 4} \right)$

2. Given $e^{3xy} = \cos(y^2)$ calculate the derivative with respect to x . (2 points)

$$\frac{d}{dx} e^{3xy} = \frac{d}{dx} \cos(y^2)$$

$$\Leftrightarrow (3y + 3xy') e^{3xy} = -2yy' \cos(y^2).$$

$$\Leftrightarrow 3xy' e^{3xy} + 2yy' \cos(y^2) = -3ye^{3xy}$$

$$\Leftrightarrow y' = \frac{-3ye^{3xy}}{3xe^{3xy} + 2y \cos(y^2)}$$

$\frac{1}{2}$ for work

3. Determine the value(s) of t for which $f(t) = -2 \cdot \sqrt{1-t^4} - \sin^{-1}(t^2)$ has a horizontal tangent line. (2 points)

Solve $f'(t) = 0$ for t .

(Solution similar to form A)

1. Find the derivative of $f(x) = x^{3x} \cdot \cos^6(x^3 + 2)$. (2 points)

$$\ln f = \ln [x^{3x} \cdot \cos^6(x^3 + 2)] = 3x \ln x + 6 \ln \cos(x^3 + 2) \left(\frac{1}{2}\right)$$

So $\frac{1}{f} \cdot f' = 3 \ln x + 3x \left(\frac{1}{x}\right) + 6 \frac{-3x^2 \sin(x^3 + 2)}{\cos(x^3 + 2)}$

$$\rightarrow f' = x^{3x} \cos^6(x^3 + 2) \left(3 \ln x + 3 - \frac{18x^2 \sin(x^3 + 2)}{\cos(x^3 + 2)}\right)$$

2. Given $e^{5xy} = \sin(y^2)$ calculate the derivative with respect to x . (2 points)

$$\frac{d}{dx} e^{5xy} = \frac{d}{dx} \sin(y^2)$$

$$\Leftrightarrow (5x + 5xy') e^{5xy} = 2yy' \cos(y^2)$$

$$\Leftrightarrow 5x e^{5xy} = -5xy' e^{5xy} + 2yy' \cos(y^2)$$

$$\Leftrightarrow y' = \frac{5x e^{5xy}}{2y \cos(y^2) - 5x e^{5xy}}$$

$\frac{1}{2}$ for work

3. Determine the value(s) of t for which $f(t) = -2 \cdot \sqrt{1-t^4} + \cos^{-1}(t^2)$ has a horizontal tangent line. (2 points)

Solve $f(t) = 0$ for t .

(Solution similar to form B)