

Quiz 8 - Problem References

MAC 2311

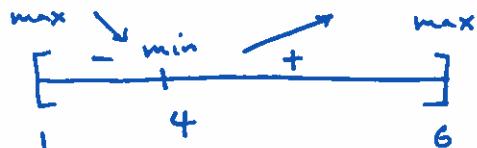
1. Section 4.2 #31, 33.
2. Section 4.3 #4, 5 and Webassign HW 10 #5.
3. Section 4.2 #67, 68.

Show your work to earn full credit.

1. Determine the coordinates of the minimum and maximum values of $f(x) = x^3 - 6x^2 + 8$ on the interval $[1, 6]$. (2 points)

 $[1, 6]$

Solve $f'(x) = 0$ for x : $f'(x) = 3x(x-4) = 0 \Rightarrow x=0, 4$.



$$f'(x) \underset{\frac{1}{2}}{}$$

maximum: $f(1) = 3 \Rightarrow$ max coords: $(1, 3)$

 $\frac{1}{2}$

minimum: $f(4) = -24 \Rightarrow$ min coords: $(4, -24)$

maximum: $f(6) = 8 \Rightarrow$ max coords: $(6, 8)$

2. Find a point c satisfying the conclusion of the Mean Value Theorem for $f(x) = \frac{x}{x+4}$ on the interval $[1, 8]$. (2 points)

$f(8) = \frac{8}{12} = \frac{2}{3}$, $f(1) = \frac{1}{5}$. So there exists c in $(1, 8)$

such that $\frac{1}{2} f'(c) = \frac{f(8) - f(1)}{8-1} = \frac{\frac{2}{3} - \frac{1}{5}}{7} = \frac{10-3}{105} = \frac{1}{15}$.

Since $f'(x) = \frac{(x+4) - x}{(x+4)^2} = \frac{4}{(x+4)^2}$, we have that

$$(c+4)^2 = 60 \Rightarrow c = 2\sqrt{15} - 4$$

3. Given $f(x) = \frac{x^2}{8x-15}$ and $g(x) = x + x^{-1}$, determine which of these two functions satisfies the initial conditions of Rolle's Theorem on the interval $[3, 5]$. Then, find a point c satisfying the conclusion of the Rolle's Theorem. (2 points)

Same as Form B.

1. Determine the coordinates of the minimum and maximum values of $f(x) = x^3 - 6x^2 + 8$ on the interval $[-1, 4]$. (2 points)

Solve $f'(x) = 0$ for x :

$$f'(x) = 3x^2 - 12x = 3x(x-4) = 0 \Rightarrow x=0, 4. \quad \frac{1}{2}$$

maximum: $f(0) = 0^3 - 6(0)^2 + 8 = 8 \Rightarrow \text{max coords: } (0, 8)$

minimum: $f(-1) = -1 - 6 + 8 = 1 \Rightarrow \text{min coords: } (-1, 1)$

minimum: $f(4) = 64 - 6(16) + 8 = -24 \Rightarrow \text{min coords: } (4, -24)$

2. Find a point c satisfying the conclusion of the Mean Value Theorem for $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$. (2 points)

$$f(4) = \frac{4}{6} = \frac{2}{3}, \quad f(1) = \frac{1}{3}$$

So there exists c in $(1, 4)$ such that

$$\frac{1}{2} f'(c) = \frac{f(4) - f(1)}{4 - 1} = \frac{\frac{2}{3} - \frac{1}{3}}{3} = \frac{1}{9} \quad \frac{1}{2}$$

Since $f'(x) = \frac{(x+2) - x}{(x+2)^2} = \frac{2}{(x+2)^2}$, we have that

$$(c+2)^2 = 18 \Rightarrow c = 3\sqrt{2} - 2 \quad \frac{1}{2}$$

3. Given $f(x) = \frac{x^2}{8x-15}$ and $g(x) = x + x^{-1}$, determine which of these two functions satisfies the initial conditions of Rolle's Theorem on the interval $[3, 5]$. Then, find a point c satisfying the conclusion of the Rolle's Theorem. (2 points)

Since $f(3) = \frac{9}{24-15} = 1$ and $f(5) = \frac{25}{40-15} = 1$, $f(x)$ satisfies Rolle's

Theorem on $[3, 5]$. So there is a value c in $(3, 5)$ such that $f'(c) = 0$.

$$f'(x) = \frac{2x(8x-15) - 8(x^2)}{(8x-15)^2} = \frac{8x^2 - 30x}{(8x-15)^2} = \frac{2x(4x-15)}{(8x-15)^2} \quad \frac{1}{2}$$

So $f'(0) = 0$ and $f'(\frac{15}{4}) = 0$. Hence $c = \frac{15}{4}$ because c is in $(3, 5)$.

$\frac{1}{2}$ work

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Name: key

Quiz 8 - Form C

November 3, 2015

Show your work to earn full credit.

1. Determine the coordinates of the minimum and maximum values of $f(x) = x^3 - 6x^2 + 8$ on the interval $[1, 6]$. (2 points)

Solve $f'(x)=0$ for x :

$$\max \quad f'(x) = 3x^2 - 12x = 3x(x-4) = 0 \Rightarrow x=0, 4. \quad \left(\frac{1}{2}\right)$$

$$\text{maximum: } f(1) = 1^3 - 6(1)^2 + 8 = 3 \Rightarrow \text{max coords: } (1, 3)$$

$$\text{minimum: } f(4) = 64 - 6(16) + 8 = -24 \Rightarrow \text{min coords: } \cancel{(4, -24)} \quad \left(\frac{1}{2}\right)$$

$$\text{maximum: } f(6) = 6^3 - 6(6)^2 + 8 = 8 \Rightarrow \text{max coords: } (6, 8)$$

2. Find a point c satisfying the conclusion of the Mean Value Theorem for $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$. (2 points)

Same as form B.

3. Given $f(x) = \cos(x)$ and $g(x) = \sin^2(x) - \cos^2(x)$, determine which of these two functions satisfies the initial conditions of Rolle's Theorem on the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$. Then, find a point c satisfying the conclusion of the Rolle's Theorem. (2 points)

Since $g\left(\frac{\pi}{4}\right) = \sin^2\left(\frac{\pi}{4}\right) - \cos^2\left(\frac{\pi}{4}\right) = 0$ and $g\left(\frac{3\pi}{4}\right) = \sin^2\left(\frac{3\pi}{4}\right) - \cos^2\left(\frac{3\pi}{4}\right) = 0$

Since $g(x)$ satisfies Rolle's Thm on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$. So there is a value c in $(\frac{\pi}{4}, \frac{3\pi}{4})$ such that $g'(c) = 0$.

$$g'(x) = 2\sin x \cos x - 2\cos x (-\sin x) = 4\cos x \sin x \quad \left(\frac{1}{2}\right)$$

Since $\cos\left(\frac{\pi}{2}\right) = 0$, $g'\left(\frac{\pi}{2}\right) = 0$. Since $\frac{\pi}{2}$ is in $(\frac{\pi}{4}, \frac{3\pi}{4})$, $c = \frac{\pi}{2}$.

$\frac{1}{2}$ for work