

Quiz 9 - Problem References

MAC 2311

1. Section 4.3 #31, 35, 37, 39.
2. Section 4.4 #7, 13 and Webassign HW 11 #5.
3. Section 4.4 #33, 35, 37.

MAC 2311 - Period: _____
 Quiz 9 - Form A
 November 10, 2015

Name: Key
 Show your work to earn full credit.

1. Find the critical points and the intervals on which $\frac{2}{x^2+1}$ is increasing or decreasing. Use the first derivative test to determine whether each critical point is a local minimum or maximum (or neither). (2 points)

Let $f(x) = \frac{2}{x^2+1}$. Then $f'(x) = \frac{-4x}{(x^2+1)^2}$.

Solve $f'(x) = 0 \Leftrightarrow -4x = 0 \Leftrightarrow x = 0$

A horizontal number line with arrows at both ends. It has three tick marks: a plus sign (+) on the left, a zero (0) in the middle, and a minus sign (-) on the right. Arrows point from the plus sign to the zero, and from the zero to the minus sign.

$\left(\frac{1}{2}\right)$ Increasing: $(-\infty, 0)$
 $\left(\frac{1}{2}\right)$ Decreasing: $(0, \infty)$

$\left(\frac{1}{2}\right)$ Maximum value @ $x=0$

2. Determine the intervals on which $f(x) = x^2 + 8 \ln x - 15x$ is concave up/down and list any points of inflection. (2 points)

$f'(x) = 2x + \frac{8}{x} - 15$ $f''(x) = 2 - \frac{8}{x^2} = \frac{2(x^2-4)}{x^2}$

$f''(x) = 0$ when $x = \pm 2$ and $f''(x)$ is undefined when $x=0$.
 Since the domain of $f(x)$ is $(0, \infty)$ we only need to consider $x=2$.

$\begin{array}{c} - \\ \hline 0 & 2 \\ + \end{array} \rightarrow f''$

$\left(\frac{1}{2}\right)$ Concave up: $(2, \infty)$
 $\left(\frac{1}{2}\right)$ Concave down: $(0, 2)$

$\left(\frac{1}{2}\right)$ Inflection pt @ $x=2$

3. Find the critical points and apply the Second Derivative Test given $f(x) = xe^{-x^2}$. (2 points)

$f'(x) = e^{-x^2} - 2x^2e^{-x^2} = \frac{1-2x^2}{e^{x^2}}$. So $f'(x)=0$ when $x = \pm \frac{1}{\sqrt{2}}$.

$f''(x) = -2xe^{-x^2} - 4x^2e^{-x^2} + 4x^3e^{-x^2} = \frac{2x(2x^2-3)}{e^{x^2}}$

Note that $2\left(\pm \frac{1}{\sqrt{2}}\right)^2 - 3 = 1 - 3 = -2$.

$\left(\frac{1}{2}\right)$ for work.

So $f''\left(\frac{1}{\sqrt{2}}\right) = \frac{-4\left(\frac{1}{\sqrt{2}}\right)}{e^{1/2}} < 0$ and $f''\left(-\frac{1}{\sqrt{2}}\right) > 0$. $\left(\frac{1}{2}\right)$ f has min @ $x = -\frac{1}{\sqrt{2}}$, max @ $x = \frac{1}{\sqrt{2}}$.

MAC 2311 - Period: _____
Quiz 9 - Form B
November 10, 2015

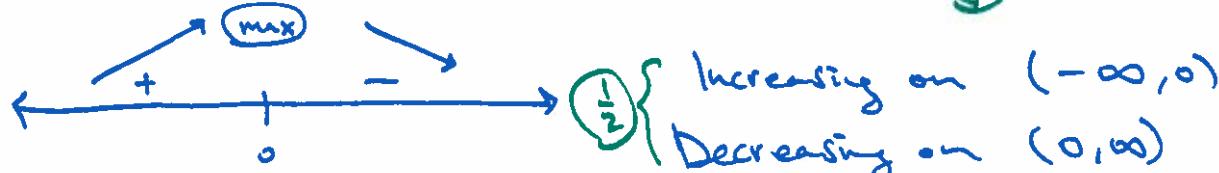
Name: Key

Show your work to earn full credit.

1. Find the critical points and the intervals on which $\frac{5}{x^2+1}$ is increasing or decreasing. Use the first derivative test to determine whether each critical point is a local minimum or maximum (or neither). (2 points)

Let $f(x) = \frac{5}{x^2+1}$. Then $f'(x) = \frac{-10x}{(x^2+1)^2}$ $\left(\frac{1}{2}\right)$

Solve $f'(x) = 0 \Leftrightarrow -10x = 0 \Leftrightarrow x = 0 \left(\frac{1}{2}\right)$



$\left(\frac{1}{2}\right)$ Maximum value @ $x=0$.

2. Determine the intervals on which $f(x) = x^2 + 18 \ln x - 12x$ is concave up/down and list any points of inflection. (2 points)

$$f'(x) = 2x + \frac{18}{x} - 12 \left(\frac{1}{2}\right), \quad f''(x) = 2 - \frac{18}{x^2} = \frac{2(x^2-9)}{x^2} \left(\frac{1}{2}\right)$$

$f''(x) = 0$ when $x = \pm 3$ and $f''(x)$ is undefined when $x = 0$.

Since the domain of $f(x)$ is $(0, \infty)$ we only need to consider $x = 3$



$\left(\frac{1}{2}\right)$ Inflection point @ $x = 3$.

3. Find the critical points and apply the Second Derivative Test given $f(x) = xe^{-x^2}$. (2 points)

Same as form A.

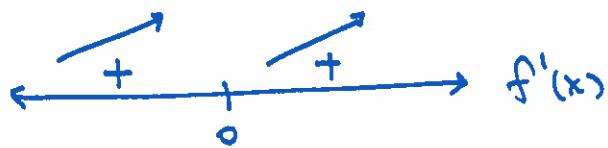
MAC 2311 - Period: _____
Quiz 9 - Form C
November 10, 2015

Name: Key _____
Show your work to earn full credit.

1. Find the critical points and the intervals on which $\frac{x^3}{x^2+1}$ is increasing or decreasing. Use the first derivative test to determine whether each critical point is a local minimum or maximum (or neither). (2 points)

Let $f(x) = \frac{x^3}{x^2+1}$. Then $f'(x) = \frac{3x^2(x^2+1) - x^3 \cdot 2x}{(x^2+1)^2} = \frac{x^4 + 3x^2}{(x^2+1)^2}$ 1/2

Solve $f'(x) = 0 \Leftrightarrow x^4 + 3x^2 = 0 \Leftrightarrow x = 0$ 1/2



1/2 Increasing on $(-\infty, 0) \cup (0, \infty)$

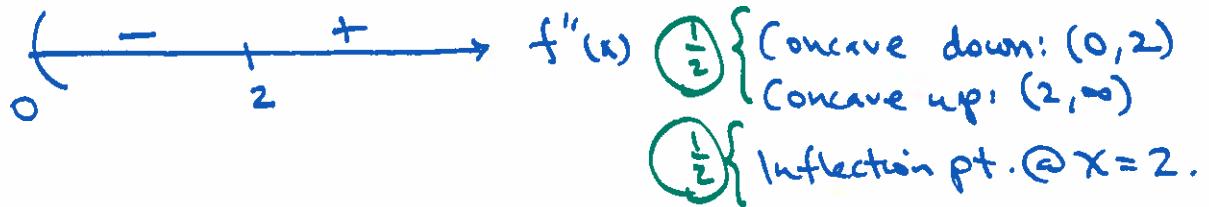
1/2 No relative max/min.

2. Determine the intervals on which $f(x) = \frac{1}{2}x^2 + 4 \ln x - 15x$ is concave up/down and list any points of inflection. (2 points)

$f'(x) = x + \frac{4}{x} - 15$ 1/2, $f''(x) = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2}$ 1/2

$f''(x) = 0$ when $x = \pm 2$ and $f''(x)$ is undefined when $x = 0$.

Since the domain of $f(x)$ is $(0, \infty)$ we only consider $x = 2$.



1/2 { Concave down: $(0, 2)$
Concave up: $(2, \infty)$

1/2 Inflection pt. @ $x = 2$.

3. Find the critical points and apply the Second Derivative Test given $f(x) = xe^{-x^2}$. (2 points)

Same as form A.

1. Find the critical points and the intervals on which $\frac{x^3}{x^2 + 1}$ is increasing or decreasing. Use the first derivative test to determine whether each critical point is a local minimum or maximum (or neither). (2 points)

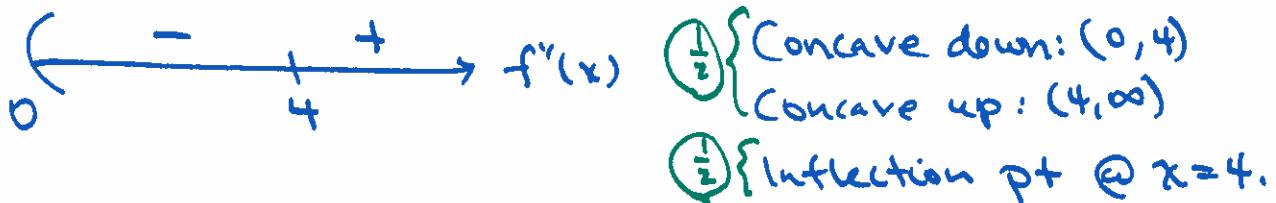
Same as form C

2. Determine the intervals on which $f(x) = \frac{1}{2}x^2 + 16 \ln x - 9x$ is concave up/down and list any points of inflection. (2 points)

$$f'(x) = x + \frac{16}{x} - 9 \quad \text{(circle)} \quad f''(x) = 1 - \frac{16}{x^2} = \frac{x^2 - 16}{x^2} \quad \text{(circle)}$$

$f''(x) = 0$ when $x = \pm 4$ and $f''(x)$ is undefined when $x = 0$.

Since the domain of f is $(0, \infty)$ we only consider $x = 4$.



3. Find the critical points and apply the Second Derivative Test given $f(x) = xe^{-x^2}$. (2 points)

Same as form A.