MAC 2313 - Period: ____
Quiz 1
January 17, 2019

Name: KEY

**Show your work to earn full credit.**

**1.** Find an equation of the set of all points equidistant from the points A(3, 0, 3) and B(0, 1, 2). (*2 points*)

Let $P(x,y,z)$ be an arbitrary point. we want $|AP| = |BP|$.

$$|AP|^2 = (x-3)^2 + y^2 + (z-3)^2 = x^2 - 6x + 9 + y^2 + z^2 - 6z + 9$$

$$|BP|^2 = (x-0)^2 + (y-1)^2 + (z-2)^2 = x^2 + y^2 - 2y + 1 + z^2 - 4z + 4$$

So $0 = |AP|^2 - |BP|^2 = -6x + 2y - 2z + 13$

$\iff$ $\boxed{6x - 2y + 2z = 13}$ OR $\boxed{-6x + 2y - 2z = -13}$

**2.** Find a unit vector that is orthogonal to both $\boldsymbol{i} + 2\boldsymbol{j}$ and $2\boldsymbol{j} + \boldsymbol{k}$. (*3 points*)

You can use either dot product or cross product to solve this problem

using cross product (faster method):

$$\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} = (2-0)\hat{i} - (1-0)\hat{j} + (2-0)\hat{k} = \langle 2, -1, 2 \rangle$$

Need to normalize to make unit vector:

$$|\vec{u}| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{9} = 3 \longrightarrow \hat{u} = \frac{\vec{u}}{|\vec{u}|} = \boxed{\frac{1}{3}\langle 2, -1, 2 \rangle}$$

Note: Also $\frac{1}{3}\langle -2, 1, -2 \rangle$ (still orthogonal), opposite direction)

**3.** State the formula for $\text{proj}_{\boldsymbol{v}}\boldsymbol{u}$. Then compute $\text{proj}_{\boldsymbol{v}}\boldsymbol{u}$ for the vectors $\boldsymbol{u} = \langle 3, -1, 1 \rangle$ and $\boldsymbol{v} = \langle 4, 7, -4 \rangle$. (*3 points*)

$$\boxed{\text{proj}_v u = \left( \frac{u \cdot v}{|v|^2} \right) v}$$

Computing required scalars for formula:

$$u \cdot v = \langle 3, -1, 1 \rangle \cdot \langle 4, 7, -4 \rangle = 12 - 7 - 4 = 1$$

$$|v|^2 = (4)^2 + (7)^2 + (-4)^2 = 16 + 49 + 16 = 81$$

So $\boxed{\text{proj}_v u = \frac{1}{81} \langle 4, 7, -4 \rangle}$

Problem References:

1. MAC2313 L1 HW Assignment Problem #15 and L1 Notes NYTI #3. Answer: The plane $6x - 2y + 2z = 13$.
2. MAC2313 L3 HW Assignment Problem #9 and L3 Notes NYTI #1. Answer: $\pm\frac{1}{3}\langle -2, 1, -2\rangle$
3. MAC2313 L3 HW Assignment Problem #11. Answer: $\text{proj}_{\boldsymbol{v}}\boldsymbol{u} = \left(\frac{\boldsymbol{u}\cdot\boldsymbol{v}}{|\boldsymbol{v}|^2}\right)\boldsymbol{v} = \frac{1}{81}\langle 4, 7, -4\rangle$