1. Find an equation of the set of all points equidistant from the points A(3, 0, 3) and B(0, 1, 2). (2 points)

Let \( P(x, y, z) \) be an arbitrary point. We want \( |AP| = |BP| \).

\[
|AP|^2 = (x-3)^2 + y^2 + (z-3)^2 = x^2 - 6x + 9 + y^2 + z^2 - 6z + 9
\]

\[
|BP|^2 = (x-0)^2 + (y-1)^2 + (z-2)^2 = x^2 + y^2 - 2y + 1 + z^2 - 4z + 4
\]

So \( 0 = (AP)^2 - |BP|^2 = -6x + 2y - 2z + 13 \)

\[\Rightarrow \quad 6x - 2y + 2z = 13 \quad \text{or} \quad -6x + 2y - 2z = -13 \]

2. Find a unit vector that is orthogonal to both \( i + 2j \) and \( 2j + k \). (3 points)

You can use either dot product or cross product to solve this problem using cross product (faster method):

\[
\mathbf{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} = \begin{pmatrix} 2-0 \end{pmatrix} \hat{i} - \begin{pmatrix} 1-0 \end{pmatrix} \hat{j} + \begin{pmatrix} 2-0 \end{pmatrix} \hat{k} = \langle 2, -1, 2 \rangle
\]

Need to normalize to make unit vector:

\[
|\mathbf{u}| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{9} = 3 \quad \Rightarrow \quad \hat{u} = \frac{\mathbf{u}}{|\mathbf{u}|} = \begin{pmatrix} \frac{2}{3} \end{pmatrix} \langle 2, -1, 2 \rangle
\]

Note: Also \( \frac{1}{3} \langle -2, 1, -2 \rangle \) (still orthogonal, opposite direction)

3. State the formula for proj\(_v\) u. Then compute proj\(_v\) u for the vectors \( u = (3, -1, 1) \) and \( v = (4, 7, -4) \). (3 points)

\[
\text{proj}_v u = \left( \frac{u \cdot v}{|v|^2} \right) v
\]

Computing required scalars for formula:

\[
u \cdot v = \langle 3, -1, 1 \rangle \cdot \langle 4, 7, -4 \rangle = 12 - 7 - 4 = 1 \]

\[
|v|^2 = (4)^2 + (7)^2 + (-4)^2 = 16 + 49 + 16 = 81
\]

\[
\Rightarrow \quad \text{proj}_v u = \frac{1}{81} \langle 4, 7, -4 \rangle
\]
Problem References:

1. MAC2313 L1 HW Assignment Problem #10 and L1 Notes NYTI #3. Answer: The plane $6x - 2y + 2z = 13$. 
2. MAC2313 L3 HW Assignment Problem #9 and L3 Notes NYTI #1. Answer: $\pm \frac{1}{3}(-2, 1, -2)$
3. MAC2313 L3 HW Assignment Problem #11. Answer: $\proj_u v = \left( \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{v}||^2} \right) \mathbf{v} = \frac{1}{81}(4, 7, -4)$