

UF MATH CIRCLE: SPROUTS AND BRUSSELS SPROUTS

1. SPROUTS

1.1. **The Game.** Sprouts is a two-player game invented by mathematicians John Conway and Michael Paterson: all you need is a pencil and paper (or whiteboard and marker).

Setup: Draw two dots (later you can explore with starting with more than two dots).

Rules: Players alternate taking turns. Each turn consists of drawing a line from one dot to another dot or to itself and then adding a new dot on that newly constructed line. When drawing new lines they must follow these rules:

- (1) Lines can be curvy, but may not cross itself or touch another line or dot except at the start and end.
- (2) No dot can have more than three lines emanating from it (a line starting and ending at the same dot counts as two).

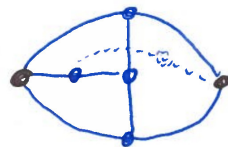
Objective: The last player to make a legal move wins!



Allowed



Not Allowed



Not allowed

1.2. **Questions.** After playing some games, you should think about the mathematics of sprouts. Here are a few questions to get you started.

- Does the first or second player win? What if you change the number of starting dots?
- Are there any tactics that you can use to help you win?
- How long does a game of sprouts go on for? What is the shortest or longest possible game if you start with a given number of dots?
- Is there any pattern in the number of faces at the end of a game? (A face is a region bounded by lines. We also count the region outside of any of the lines as a face.)
- What other patterns do you notice?
- What interesting variations can you make to the rules and setup?

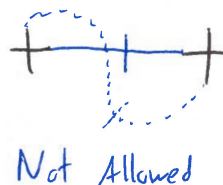
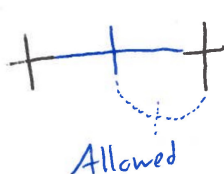
2. BRUSSELS SPROUTS

2.1. **The Game.** Brussels Sprouts is a seemingly similar game to Sprouts.

Setup: Draw two plus signs $+$ (later you can explore with starting with more than two plus signs).

Rules: Players alternate taking turns. Each turn consists of drawing a line starting from an arm of a $+$ and ending at the arm of another $+$. Then you add a dash in the middle of the newly drawn line, forming a new plus sign. The new lines are allowed to be curvy, but may not cross itself, or touch another line or $+$ except at the start and end.

Objective: The last player to make a legal move wins!



2.2. **Questions.** Try playing a game of Brussels Sprouts where you keep track of how many $+$'s, how many lines, how many faces you have after each move.

Number of $+$	Number of Lines	Number of Faces

Now ask and answer your own questions about Brussels Sprouts.