Horizontal vs. Slant Asymptotes

Let $f(x) = \dfrac{N(x)}{D(x)} = \dfrac{a_n x^n + \dots + a_0}{b_m x^m + \dots + b_0}$.

Case 1: If $n < m$ (i.e., the degree of the numerator is less than the degree of the denominator), then f has $\boxed{y = 0}$ as a $\boxed{\text{horizontal}}$ asymptote.

Ex: $f(x) = \dfrac{x^2 + 2x + 5}{x^3 - x + 1}$ has $y = 0$ as a horizontal asymptote.

Case 2: If $n = m$, then f has $\boxed{y = \dfrac{a_n}{b_m}}$ as a $\boxed{\text{horizontal}}$ asymptote.

$\dfrac{a_n}{b_m} \to$ ratio of leading coefficients

Ex: $f(x) = \dfrac{2x^2 + 5x - 1}{3x^2 - x + 2}$ has $y = \dfrac{2}{3}$ as a horizontal asymptote.

Case 3: If $n > m$, then f has $\boxed{\text{no horizontal}}$ asymptote.

However, if the degree of the numerator is one more than the degree of the denominator, then f has a slant asymptote; use long division, and the slant asymptote will be the $\boxed{\text{quotient}}$.

Ex: $f(x) = \dfrac{x^2 - x}{x + 1}$ has no horizontal asymptote.

$$
\begin{array}{r@{}l}
 & \boxed{x - 2} + \dfrac{2}{x+1} \\
x + 1 \,\big) & x^2 - x \\
 & -(x^2 + x) \\ \hline
 & -2x \\
 & -(-2x - 2) \\ \hline
 & 2
\end{array}
$$

$\longrightarrow y = x - 2$ is the slant asymptote.