

NAME: Solution

MAC 1147 Section 3077
Quiz Ten

Please show all of your work in a NEAT and ORGANIZED fashion.

1. (3 points) Factor the following trigonometric expression:

$$\begin{aligned}\cot^2(x) + \csc(x) - 1 &= \\ \csc^2 x - 1 + \csc x - 1 &= \\ \csc^2 x + \csc x - 2 &= \\ (\csc x + 2)(\csc x - 1)\end{aligned}$$

$$\begin{aligned}1 + \cot^2 x &= \csc^2 x \\ \cot^2 x &= \csc^2 x - 1\end{aligned}$$

2. (3 points) Verify the following identity:

$$\begin{aligned}\cos t(\csc^2 t - 1) &= \frac{\cos^3 t}{\sin^2 t} \\ \cos t(1 + \cot^2 t - 1) &= \\ \cos t(\cot^2 t) &= \\ \cos t\left(\frac{\cos^2 t}{\sin^2 t}\right) &= \\ \frac{\cos^3 t}{\sin^2 t} &\checkmark\end{aligned}$$

3. (3 points) Find all solutions of the equation in the interval $[0, 2\pi)$:

$$\begin{aligned}2\cos^2 x - \sin x - 1 &= 0 \\ 2(1 - \sin^2 x) - \sin x - 1 &= 0 \\ 2 - 2\sin^2 x - \sin x - 1 &= 0 \\ -2\sin^2 x - \sin x + 1 &= 0 \\ 2\sin^2 x + \sin x - 1 &= 0 \\ (2\sin x - 1)(\sin x + 1) &= 0 \\ \sin x = \frac{1}{2} \text{ or } \sin x = -1\end{aligned}$$

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x\end{aligned}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}; \frac{3\pi}{2}$$