

NAME: Solution

MAC 1147 Section 3089
Quiz Ten

Please show all of your work in a NEAT and ORGANIZED fashion.

1. (3 points) Factor the following trigonometric expression:

$$\begin{aligned}\cot^2(x) + \csc(x) - 1 &= \\ \csc^2 x - 1 + \csc x - 1 &= \\ \csc^2 x + \csc x - 2 &= \\ (\csc x + 2)(\csc x - 1)\end{aligned}$$

$$\begin{aligned}1 + \cot^2 x &= \csc^2 x \\ \cot^2 x &= \csc^2 x - 1\end{aligned}$$

2. (3 points) Verify the following identity:

$$\begin{aligned}\frac{1}{\cos x + 1} + \frac{1}{\cos x - 1} &= -2 \csc x \cot x \\ \frac{\cos x - 1 + \cos x + 1}{(\cos x + 1)(\cos x - 1)} &= \\ \frac{2 \cos x}{-\sin^2 x} &= \\ -2 \left(\frac{1}{\sin x} \right) \left(\frac{\cos x}{\sin x} \right) &= \\ -2 \csc x \cot x &\quad \checkmark\end{aligned}$$

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ \cos^2 x - 1 &= -\sin^2 x\end{aligned}$$

3. (3 points) Find all solutions of the equation in the interval $[0, 2\pi)$:

$$\begin{aligned}2 \sin^2 x - \cos x - 2 &= 0 \\ 2(1 - \cos^2 x) - \cos x - 2 &= 0 \\ \cancel{2} - 2 \cos^2 x - \cos x - \cancel{2} &= 0 \\ -\cos x (2 \cos x + 1) &= 0 \\ \cos x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2} \\ x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}\end{aligned}$$

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ \sin^2 x &= 1 - \cos^2 x\end{aligned}$$