

NAME: Solution

MAC 2311 Section 3071
Quiz Two

Please show all of your work in a NEAT and ORGANIZED fashion.

1. (2 points) Evaluate the limit, if it exists. If the limit does not exist, explain why.

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{(\sqrt{16+h}-4)(\sqrt{16+h}+4)}{h(\sqrt{16+h}+4)} &= \\ \lim_{h \rightarrow 0} \frac{\cancel{16+h}-16}{h(\sqrt{16+h}+4)} &= \\ \lim_{h \rightarrow 0} \frac{1}{\sqrt{16+h}+4} &= \frac{1}{\sqrt{16}+4} = \frac{1}{8}\end{aligned}$$

2. (2 points) Evaluate the limit, if it exists. If the limit does not exist, explain why.

$$\begin{aligned}\lim_{x \rightarrow 3^+} \frac{x^2+x-12}{|x-3|} &= \lim_{x \rightarrow 3^+} \frac{(x+4)(x-3)}{x-3} = \lim_{x \rightarrow 3^+} x+4 = 7 \\ \lim_{x \rightarrow 3^-} \frac{x^2+x-12}{|x-3|} &= \lim_{x \rightarrow 3^-} \frac{(x+4)(x-3)}{-(x-3)} = \lim_{x \rightarrow 3^-} -x-4 = -7\end{aligned}$$

The left-hand and right-hand limits are not equal, so the limit does not exist.

3. (2 points) Explain why the function $f(x)$ is discontinuous at $x = 1$:

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$$

$$\begin{aligned}\lim_{x \rightarrow 1} f(x) &= \\ \lim_{x \rightarrow 1} \frac{x^2-1}{x-1} &= \\ \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} &= \\ \lim_{x \rightarrow 1} x+1 &= 2\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 1} f(x) &= 2, \text{ but } f(1) = 0. \\ \text{So } \lim_{x \rightarrow 1} f(x) &\neq f(1).\end{aligned}$$