NAME: Solution

MAC 2311 Section 3071
Quiz Seven

Please show all of your work in a NEAT and ORGANIZED fashion.

1. Let \( f(x) = x^4 - \frac{3}{2}x^2 - 36x + 2 \).

(a) (1 point) Find the intervals on which \( f \) is increasing or decreasing.

\[
  f'(x) = 3x^2 - 3x - 36 = 0
\]

\[
  x^2 - x - 12 = 0
\]

\[
  (x-4)(x+3) = 0
\]

\[
  \text{inc: } (-\infty, -3) \cup (4, \infty)
\]

\[
  \text{dec: } (-3, 4)
\]

(b) (1 point) At what values of \( x \) does \( f \) have a local maximum or local minimum? Explain.

\( f \) has a local maximum at \( x = -3 \) because \( f' \) changes from positive to negative; \( f \) has a local minimum at \( x = 4 \) because \( f' \) changes from negative to positive.

(c) (1 point) Find the intervals of concavity and any inflection points.

\[
f''(x) = 6x - 3 = 0
\]

\[
2x - 1 = 0
\]

\[
\text{concave up: } (\frac{1}{2}, \infty)
\]

\[
\text{concave down: } (-\infty, \frac{1}{2})
\]

2. (3 points) Find the limit.

\[
\lim_{x \to \infty} x e^{-x}
\]

\[
y = x e^{-x} \to \ln y = e^{-x} \ln x = \frac{\ln x}{e^x} \]

\[
\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln x}{e^x}
\]

\[
= \lim_{x \to \infty} \frac{\ln x}{e^x}
\]

\[
= \lim_{x \to \infty} \frac{1}{xe^x} = 0
\]

So, \( \lim_{x \to \infty} x e^{-x} = \lim_{x \to \infty} y = \lim_{x \to \infty} e^{\ln y} = e^0 = 1 \).