MAC 1105 Section 1A27 **SOLUTIONS**

Quiz 5A (Modules 9)

Be sure to show all your work for full credit!

1. (2 points) Solve the inequality. Express your answer in <u>interval notation</u>.

$$-7 \leq -3x$$

Divide both sides of the inequality by -3 and since we are dividing by a negative number we need to flip the inequality sign from being $\leq$ to being $\geq$,

$$\frac{-7}{-3} \geq x$$

This means that x is less than or <u>equal</u> to $7/3$. So, the interval notation is $(-\infty, \frac{7}{3}]$. Notice we have a bracket after $7/3$ because $7/3$ is included.

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2. (3 points) Solve the inequality. Express your answer in <u>interval notation</u>.

$$x^2 - 12x > -35$$

We need to get all terms to one side of the inequality and zero on the other. So, add 35 to both sides:

$$x^2 - 12x + 35 > 0$$

Now factor the left hand side of the inequality

$$(x-7)(x-5) > 0$$

Now we need to know what x-values make $(x-7)(x-5) > 0$ a true statement. So, let's make a number. First we need to know when $(x-7)(x-5) = 0$. It is 0 when $x = 5, 7$. So, 5,7 will break up our number line into intervals of positive and negative numbers.

Now we have to test the intervals $(-\infty, 5), (5, 7)$, and $(7, \infty)$. When testing these intervals, we take any number in the interval and plug it into $(x-7)(x-5)$. If we get a negative number, that means every number in the interval will give us a negative number. If we get a postive number, that means every number in the interval will give us a positive number.

So, let's first test the interval $(-\infty, 5)$. We can pick any number in this interval to test. Let's test $x = 0$. Now, we plug 0 into $(x-7)(x-5)$. So, $(0-7)(0-5) = (-7)(-5) = 35$. So, we got a positive number. That means that every number in the interval $(-\infty, 5)$, when plugged into $(x-7)(x-5)$ will give us a positive number. Now look at our statement. We want $(x-7)(x-5) > 0$. Is a positive number larger than 0? Yes, of course. So, every number in the interval $(-\infty, 5)$ makes the statement $(x-7)(x-5) > 0$ true. So, that interval is one of our answers.

Now do the same steps for the intervals $(5, 7)$ and $(7, \infty)$. When we plug $x = 6$, we get $(6-7)(6-5) = (-1)(1) = -1$ which is not bigger than zero. So, any number in the interval $(5, 7)$ does not make the statement $(x-7)(x-5) > 0$ true. So, that interval is not one of our answers. When we plug $x = 8$, we get $(8-7)(8-5) = (1)(3) = 3$ which is bigger than zero. So, any number in the interval $(7, \infty)$ makes the statement $(x-7)(x-5) > 0$ true. So, that interval is also one of our answers.

So, the final answer is $(-\infty, 5) \cup (7, \infty)$. Notice we had parantheses after the 5 and before the 7 because 5 and 7 make $(x-7)(x-5) = 0$ and we only want $(x-7)(x-5) > 0$. If we were asked to solve $(x-7)(x-5) \geq 0$, then our answer would have been $(-\infty, 5] \cup [7, \infty)$.

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3. (2 points) Solve the inequality. Express your answer in <u>interval notation</u>.

$$|4x| < 3$$

When taking terms out of an absolute value we need to have two cases, a positive and a negative case. So, we have either $4x < 3$ or $-(4x) < 3$. Evalutating the first case we divide both sides by 4 and get:

$$x < 3/4$$

Now evaluate the second case and first we must multiply both sides by -1. This also means we have to flip the inequality sign. So, $-(4x) < 3$ becomes

$$4x > -3$$

Now divide both sides by 4 and we get

$$x > -3/4$$

Combining these two inequalities we get the interval (-3/4 , 3/4) as our answer.

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4. (3 points) Solve the inequality. Express your answer in <u>interval notation</u>.

$$\frac{9-7x}{x} \geq -4$$

We need all terms on one side and 0 on the other. So, we get:

$$\frac{9-7x}{x} + 4 \geq 0$$

Now get common denomiators on the left hand side of the inequality. That means muliplying 4 by $\frac{x}{x}$.

$$\frac{9-7x}{x} + 4 \cdot \frac{x}{x} \geq 0$$

$$\frac{9-7x}{x} + \frac{4x}{x} \geq 0$$

Now, we can add the fractions together.

$$\frac{9-7x+4x}{x} \geq 0$$

$$\frac{9-3x}{x} \geq 0$$

Now find where the numerator and demonimator equals 0. The numerator equals 0 when $x = 3$ and the denominator equals zero when $x = 0$. So we plot those on our number line. Notice that we include $x = 3$ since that would make the left hand side equal to 0 and we want to know when the left hand side is greater than or equal to 0. We don't include $x = 0$ because that makes the denominator 0 and the denominator can never equal 0.

So, now test the intervals $(-\infty, 0), (0, 3)$, and $(3, \infty)$ in a similar way to how we did so in question 2. You should get that $(-\infty, 0)$ gives you negative numbers, $(0, 3)$ gives you positive numbers, and $(3, \infty)$ gives you negative numbers. Since we want to know when $\frac{9-3x}{x}$ is greater than or equal to 0, then we want numbers that make $\frac{9-3x}{x}$ positive or 0. So, our answer is $(0, 3]$.
