MAC 2311 Section 3137

Participation Activity

1. The position function of a ball thrown into the air is given by \( H(t) = 5t - 2t^2 \) where \( H(t) \) is the height of the ball above the ground after \( t \) seconds.

   (a) Find the average velocity of the ball on the interval starting with \( t = 1 \) to the time \( 0.1 \) seconds later.

   \[
   \frac{H(1.1) - H(1)}{1.1 - 1} = \frac{5(1.1) - 2(1.1)^2 - 5 + 2}{0.1} = 0.5 - 0.42 = 0.08 \times 100 = \frac{8}{10} = \frac{4}{5}
   \]

   (b) Now write an expression for the average velocity of the ball on the time from \( t = 1 \) to \( h \) seconds later, where \( h \neq 0 \).

   \[
   \frac{5(1+h) - 2(1+h)^2 - 5 + 2}{h} = \frac{5h - 2 - 4h - 2h^2 - 5 + 2}{h} = \frac{-2h^2}{h} = 1 - 2h
   \]

   (c) What happens as \( h \) gets closer and closer to 0?

   \[\lim_{h \to 0} 1 - 2h = 1\]

   (d) Find the instantaneous velocity of the ball when \( t = 1 \).

   \[\frac{\text{ft}}{\text{sec}}\]

2. Sketch an even function \( f \) such that:

   \[\lim_{x \to 0^+} f(x) = -3, \quad \lim_{x \to 4^-} f(x) = -\infty, \quad f(0) = 2\]
3. Evaluate the limit.

\[
\lim_{x \to 1^-} \ln(1 - x) = \ln(\lim_{x \to 1^-} (1 - x)) = \ln(0^-) = -\infty
\]

4. Suppose \(\lim_{x \to 5^-} f(x) = 4\) and \(\lim_{x \to 5^+} f(x) = 0\).

(a) Does \(\lim_{x \to 5} f(x)\) exist?

\[\text{NO}\]

(b) If so, what is the \(\lim_{x \to 5} f(x)\)?

(c) If not, give a reason why the limit does not exist.

\[
\text{Since} \quad \lim_{x \to 5^-} f(x) \neq \lim_{x \to 5^+} f(x)
\]

5. Sketch a graph of a function \(f\) that satisfies all of the given condition. There are many different solutions.

\[
\lim_{x \to 1^-} f(x) = -2, \quad \lim_{x \to 1^+} f(x) = 0, \quad f(1) = 3
\]