

Quiz 2 (Lectures 6,7,8)

Be sure to show your work for full credit!

1. Evaluate the following limit.

$$\lim_{x \rightarrow 0} \frac{\sqrt{3} - \sqrt{x+3}}{x} = \frac{\sqrt{3} - \sqrt{0+3}}{0} = \frac{0-0}{0} = \frac{0}{0}$$

indeterminate form

$$\lim_{x \rightarrow 0} \frac{\sqrt{3} - \sqrt{x+3}}{x} \cdot \frac{\sqrt{3} + \sqrt{x+3}}{\sqrt{3} + \sqrt{x+3}}$$

$$= \lim_{x \rightarrow 0} \frac{3 - (x+3)}{x(\sqrt{3} + \sqrt{x+3})} = \lim_{x \rightarrow 0} \frac{3 - x - 3}{x(\sqrt{3} + \sqrt{x+3})} = \lim_{x \rightarrow 0} \frac{-x}{x(\sqrt{3} + \sqrt{x+3})}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{\sqrt{3} + \sqrt{x+3}} = \frac{-1}{\sqrt{3} + \sqrt{0+3}} = \boxed{\frac{-1}{2\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{2 \cdot 3} = \frac{-\sqrt{3}}{6}$$

2. Find the set of points at which the function is continuous.

$$f(x) = \frac{1}{\ln(\ln(x))}$$

Must have:

$$\ln(\ln(x)) \neq 0$$

$$\ln(x) > 0, \text{ and } \boxed{x > 0}$$

Solve for x:

$$e^{\ln(\ln(x))} \neq e^0 = 1$$

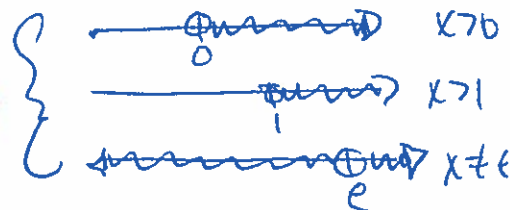
$$e^{\ln(x)} > e^0 = 1$$

$$\ln(x) \neq 1$$

$$e^{\ln(x)} \neq e^1$$

$$\boxed{x \neq e}$$

$$\boxed{x > 1}$$



These intervals intersect at:

Answer: $\boxed{(1, e) \cup (e, \infty)}$

3. Find the limit

$$\lim_{x \rightarrow \infty} \frac{2e^{2x} - e^{-2x} + 5}{-3e^{2x} + e^{-2x} - 4}$$

Divide every term by e^{2x} :

$$= \lim_{x \rightarrow \infty} \frac{2e^{2x}}{e^{2x}} - \frac{e^{-2x}}{e^{2x}} + \frac{5}{e^{2x}}$$

$$\frac{-3e^{2x}}{e^{2x}} + \frac{e^{-2x}}{e^{2x}} - \frac{4}{e^{2x}}$$

$$= \lim_{x \rightarrow \infty} \frac{2 - e^{-2x-2x}}{e^{2x}} + \frac{5}{e^{2x}}$$

$$\frac{-3 + e^{-2x-2x}}{e^{2x}} - \frac{4}{e^{2x}}$$

$$= \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{e^{4x}} + \frac{5}{e^{2x}}}{-3 + \frac{1}{e^{4x}} - \frac{4}{e^{2x}}}$$

$$\frac{2 - \frac{1}{e^{4x}} + \frac{5}{e^{2x}}}{-3 + \frac{1}{e^{4x}} - \frac{4}{e^{2x}}}$$

$$= \frac{2 - \frac{1}{e^{\infty}} + \frac{5}{e^{\infty}}}{-3 + \frac{1}{e^{\infty}} - \frac{4}{e^{\infty}}}$$

$$\frac{2 - 0 + 0}{-3 + 0 - 0}$$

$$= \frac{2 - 0 + 0}{-3 + 0 - 0}$$

$$= \frac{2}{-3}$$

$$= \boxed{-\frac{2}{3}}$$