3

Logic

What You Will Learn

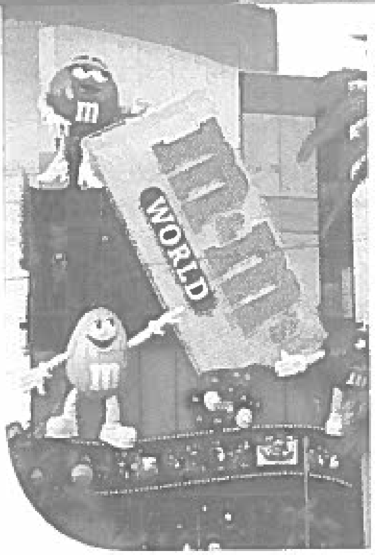
- Statements, quantifiers, and compound statements
- Statements involving the words *not, and, or, if . . . then . . .*, and *if and only if*
- Truth tables for negations, conjunctions, disjunctions, conditional statements, and biconditional statements
- Self-contradictions, tautologies, and implications
- Equivalent statements, De Morgan's laws, and variations of conditional statements
- Symbolic arguments and standard forms of arguments
- Euler diagrams and syllogistic arguments
- Using logic to analyze switching circuits

Why This is Important

The study of logic enables us to communicate effectively, make more convincing arguments, and develop patterns of reasoning for decision making. Logic is also used in the programming of modern electronic devices such as cell phones and digital cameras.

◄ *Logic is used in the programming of cell phones and digital cameras.*

SECTION 3.1 Statements and Logical Connectives



▲ *"Melts in your mouth, not in your hands." Logic symbols can be used to help us analyze statements made by advertisers.*

Advertisements often rely on spoken or written statements that are used to favorably portray the advertised product and form a convincing argument that will persuade us to purchase the product. Some familiar advertising statements are: *Don't leave home without it; It takes a licking and keeps on ticking; Sometimes you feel like a nut, sometimes you don't; When it rains it pours*; and *Melts in your mouth, not in your hands*. In this section, we will learn how to represent statements using logic symbols that may help us better understand the nature of the statement. We will use these symbols throughout the chapter to analyze more complicated statements.

Why ***This Is Important*** Statements appear in everyday life—for example, in legal documents, product instructions, and game rules in addition to advertising.

History

The ancient Greeks were the first people to systematically analyze the way humans think and arrive at conclusions. Aristotle (384–322 B.C.) organized the study of logic for the first time in a work called *Organon*. As a result of his work, Aristotle is called the father of logic. The logic from this period, called ***Aristotelian logic***, has been taught and studied for more than 2000 years.

Since Aristotle's time, the study of logic has been continued by other great philosophers and mathematicians. Gottfried Wilhelm Leibniz (1646–1716) had a deep conviction that all mathematical and scientific concepts could be derived from logic. As a result, he became the first serious student of ***symbolic logic***. One difference between symbolic logic and Aristotelian logic is that in symbolic logic, as its name implies, symbols (usually letters) represent written statements. A self-educated English mathematician, George Boole (1815–1864), is considered to be the founder of symbolic logic because of his impressive work in this area. Among Boole's publications are *The Mathematical Analysis of Logic* (1847) and *An Investigation of the Law of Thought* (1854). Mathematician Charles Dodgson, better known as Lewis Carroll, incorporated many interesting ideas from logic into his books *Alice's Adventures in Wonderland* and *Through the Looking Glass* and his other children's stories.

Logic has been studied through the ages to exercise the mind's ability to reason. Understanding logic will enable you to think clearly, communicate effectively, make more convincing arguments, and develop patterns of reasoning that will help you in making decisions. It will also help you to detect the fallacies in the reasoning or arguments of others such as advertisers and politicians. Studying logic has other practical applications, such as helping you to understand wills, contracts, and other legal documents.

The study of logic is also good preparation for other areas of mathematics. If you preview Chapter 12, on probability, you will see formulas for the probability of A or B and the probability of A and B, symbolized as $P(A \text{ or } B)$ and $P(A \text{ and } B)$, respectively. Special meanings of common words such as *or* and *and* apply to all areas of mathematics. The meaning of these and other special words is discussed in this chapter.

Logic and the English Language

In reading, writing, and speaking, we use many words such as *and*, *or*, and *if . . . then . . .* to connect thoughts. In logic we call these words ***connectives***. How are these words interpreted in daily communication? A judge announces to a convicted offender, "I hereby sentence you to five months of community service *and* a fine of $100." In this case, we normally interpret the word *and* to indicate that *both* events will take place. That is, the person must perform community service and must also pay a fine.

Now suppose a judge states, "I sentence you to six months in prison *or* 10 months of community service." In this case, we interpret the connective *or* as meaning the convicted person must either spend the time in jail or perform community service, but not both. The word *or* in this case is the ***exclusive or***. When the exclusive *or* is used, one or the other of the events can take place, but *not both*.

In a restaurant, a waiter asks, "May I interest you in a cup of soup or a sandwich?" This question offers three possibilities: You may order soup, you may order a sandwich, or you may order both soup and a sandwich. The *or* in this case is the ***inclusive or***. When the inclusive *or* is used, one or the other, *or both* events can take place. *In this chapter, when we use the word* or *in a logic statement, it will mean the inclusive* or *unless stated otherwise*.

If–then statements are often used to relate two ideas, as in the bank policy statement "If the average daily balance is greater than $500, then there will be no service charge." If–then statements are also used to emphasize a point or add humor, as in the statement "If the Cubs win, then I will be a monkey's uncle."

Now let's look at logic from a mathematical point of view.

Statements and Logical Connectives

▲ *The Brooklyn Bridge in New York City*

A sentence that can be judged either true or false is called a ***statement***. Labeling a statement true or false is called *assigning a truth value* to the statement. Here are some examples of statements.

1. The Brooklyn Bridge goes over San Francisco Bay.
2. Disney World is in Idaho.
3. The Mississippi River is the longest river in the United States.

In each case, we can say that the sentence is either true or false. Statement 1 is false because the Brooklyn Bridge does not go over San Francisco Bay. Statement 2 is false because Disney World is in Florida. By looking at a map or reading an almanac, we can determine that the Mississippi River is the longest river in the United States; therefore, statement 3 is true.

The three sentences discussed above are examples of ***simple statements*** because they convey one idea. Sentences combining two or more ideas that can be assigned a truth value are called ***compound statements***. Compound statements are discussed shortly.

Quantifiers

Sometimes it is necessary to change a statement to its opposite meaning. To do so, we use the ***negation*** of a statement. For example, the negation of the statement "Emily is at home" is "Emily is not at home." The negation of a true statement is always a false statement, and the negation of a false statement is always a true statement. We must use special caution when negating statements containing the words ***all***, ***none*** (or ***no***), and ***some***. These words are referred to as ***quantifiers***.

Consider the statement "All lakes contain fresh water." We know this statement is false because the Great Salt Lake in Utah contains salt water. Its negation must therefore be true. We may be tempted to write its negation as "No lake contains fresh water," but this statement is also false because Lake Superior contains fresh water. Therefore, "No lakes contain fresh water" is not the negation of "All lakes contain fresh water." The correct negation of "All lakes contain fresh water" is "Not all lakes contain fresh water" or "At least one lake does not contain fresh water" or "Some lakes do not contain fresh water." These statements all imply that at least one lake does not contain fresh water, which is a true statement.

Now consider the statement "No birds can swim." This statement is false because at least one bird, the penguin, can swim. Therefore, the negation of this statement must be true. We may be tempted to write the negation as "All birds can swim," but because this statement is also false it cannot be the negation. The correct negation of the statement is "Some birds can swim" or "At least one bird can swim," each of which is a true statement.

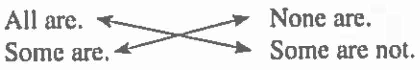
Now let's consider statements involving the quantifier *some*, as in "Some students have a driver's license." This statement is true, meaning that at least one student has a driver's license. The negation of this statement must therefore be false. The negation is "No student has a driver's license," which is a false statement.

Consider the statement "Some students do not ride motorcycles." This statement is true because it means "At least one student does not ride a motorcycle." The negation of this statement must therefore be false. The negation is "All students ride motorcycles," which is a false statement.

The negation of quantified statements is summarized as follows:

Form of statement	Form of negation
All are.	Some are not.
None are.	Some are.
Some are.	None are.
Some are not.	All are.

The following diagram might help you to remember the statements and their negations:



The quantifiers diagonally opposite each other are the negations of each other.

Example 1 *Write Negations*

Write the negation of each statement.

a) Some telephones can take photographs.

b) All houses have two stories.

Solution

a) Since *some* means "at least one," the statement "Some telephones can take photographs" is the same as "At least one telephone can take photographs." Because it is a true statement, its negation must be false. The negation is "No telephones can take photographs," which is a false statement.

b) The statement "All houses have two stories" is a false statement, since some houses have one story, some have three stories, and some may have more than three stories. Its negation must therefore be true. The negation may be written as "Some houses do not have two stories" or "Not all houses have two stories" or "At least one house does not have two stories." Each of these statements is true. ■

Compound Statements

Statements consisting of two or more simple statements are called **compound statements**. The connectives often used to join two simple statements are

and, **or,** **if, ... then ...,** **if and only if**

In addition, we consider a simple statement that has been negated to be a compound statement. The word *not* is generally used to negate a statement.

Did You Know?

Playing on Words

George Boole, Augustus De Morgan, and other mathematicians of the nineteenth century were anxious to make logic an abstract science that would operate like algebra but be applicable to all fields. One of the problems logicians faced was that verbal language could be ambiguous and could easily lead to confusion and contradiction. Comedians Bud Abbott and Lou Costello had fun with the ambiguity of language in their skit about the baseball players: "Who's on first, What's on second, I Don't Know is on third—Yeah, but who's on first?"

RECREATIONAL MATH

Sudoku

	1		4	8		5	6	
5					9	8		
	3				1	4		7
8	2			9		1		
6			1		4			9
		3		6			4	5
9		1	5				2	
		7	2					4
	5	2		7	8		3	

Solving puzzles requires us to use logic. Sudoku is a puzzle that originated in Japan and continues to gain popularity worldwide. To solve the puzzle, you need to place every digit from 1 to 9 exactly one time in each row, in each column, and in each of the nine 3 by 3 boxes. For more information and a daily puzzle see www.websudoku.com. The solution to the puzzle above can be found in the Answers section in the back of this book. For an additional puzzle see Exercise 86 on page 104.

To reduce the amount of writing in logic, it is common to represent each simple statement with a lowercase letter. For example, suppose we are discussing the simple statement "Leland is a farmer." Instead of writing "Leland is a farmer" over and over again, we can let p represent the statement "Leland is a farmer." Thereafter we can simply refer to the statement with the letter p. It is customary to use the letters p, q, r, and s to represent simple statements, but other letters may be used instead. Let's now look at the connectives used to make compound statements.

Not Statements

The negation is symbolized by $\sim$ and read "not." For example, the negation of the statement "Steve is a college student" is "Steve is not a college student." If p represents the simple statement "Steve is a college student," then $\sim p$ represents the compound statement "Steve is not a college student." For any statement p, $\sim(\sim p) \equiv p$. For example, the negation of the statement "Steve is not a college student" is "Steve is a college student."

Consider the statement "Inga is not at home." This statement contains the word *not*, which indicates that it is a negation. To write this statement symbolically, we let p represent "Inga *is* at home." Then $\sim p$ would be "Inga is not at home." *We will use this convention of letting letters such as p, q, or r represent statements that are not negated. We will represent negated statements with the negation symbol,* $\sim$.

And Statements

The ***conjunction*** is symbolized by $\wedge$ and read "and." The $\wedge$ looks like an A (for And) with the bar missing. Let p and q represent the simple statements.

p: You will perform 5 months of community service.
q: You will pay a $100 fine.

Then the following is the conjunction written in symbolic form.

$$\underbrace{\text{You will perform 5 months of community service}}_{\substack{\uparrow \\ p}} \quad \underbrace{\text{and}}_{\substack{\uparrow \\ \wedge}} \quad \underbrace{\text{you will pay a \$100 fine.}}_{\substack{\uparrow \\ q.}}$$

The conjunction is generally expressed as *and*. Other words sometimes used to express a conjunction are *but*, *however*, and *nevertheless*.

Example 2 *Write a Conjunction*

Write the following conjunction in symbolic form.

Green Day is not on tour, but Green Day is recording a new CD.

Solution Let t and r represent the simple statements.

t: Green Day is on tour.
r: Green Day is recording a new CD.

In symbolic form, the compound statement is $\sim t \wedge r$. ■

In Example 2, the compound statement is "Green Day is not on tour, but Green Day is recording a new CD." This statement could also be represented as "Green Day is not on tour, but *they* are recording a new CD." In this problem, it should be clear the word *they* means *Green Day*. Therefore, the statement "Green Day is not on tour, but they are recording a new CD" would also be symbolized as $\sim t \wedge r$.

Or Statements

The *disjunction* is symbolized by ∨ and read "or." The *or* we use in this book (except where indicated in the exercise sets) is the *inclusive or* described on page 95.

Example 3 *Write a Disjunction*

Let

p: Maria will go to the circus.
q: Maria will go to the zoo.

Write the following statements in symbolic form.

a) Maria will go to the circus or Maria will go to the zoo.
b) Maria will go to the zoo or Maria will not go to the circus.
c) Maria will not go to the circus or Maria will not go to the zoo.

Solution

a) $p \vee q$ b) $q \vee \sim p$ c) $\sim p \vee \sim q$ ■

Because *or* represents the *inclusive or,* the statement "Maria will go to the circus or Maria will go to the zoo" in Example 3(a) may mean that Maria will go to the circus, or that Maria will go to the zoo, or that Maria will go to both the circus *and* the zoo. The statement in Example 3(a) could also be written as "Maria will go to the circus or the zoo."

When a compound statement contains more than one connective, a comma can be used to indicate which simple statements are to be grouped together. When we write the compound statement symbolically, ***the simple statements on the same side of the comma are to be grouped together within parentheses.***

For example, "Pink is a singer (p) or Jennifer Aniston is an actress (j), and Dallas is in Texas (d)" is written $(p \vee j) \wedge d$. Note that the p and j are both on the same side of the comma in the written statement. They are therefore grouped together within parentheses. The statement "Pink is a singer, or Jennifer Aniston is an actress and Dallas is in Texas" is written $p \vee (j \wedge d)$. In this case, j and d are on the same side of the comma and are therefore grouped together within parentheses.

Example 4 *Understand How Commas Are Used to Group Statements*

Let

p: Dinner includes soup.
q: Dinner includes salad.
r: Dinner includes the vegetable of the day.

Write the following statements in symbolic form.

a) Dinner includes soup, and salad or the vegetable of the day.
b) Dinner includes soup and salad, or the vegetable of the day.

Solution

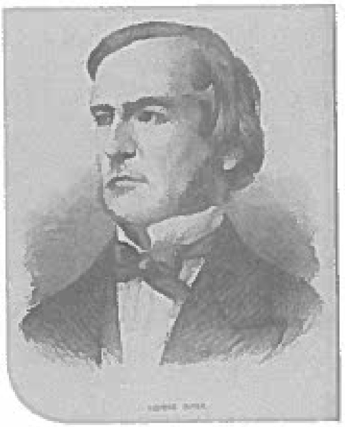
a) The comma tells us to group the statement "Dinner includes salad" with the statement "Dinner includes the vegetable of the day." Note that both statements are on the same side of the comma. The statement in symbolic form is $p \wedge (q \vee r)$.

In mathematics, we always evaluate the information within the parentheses first. Since the conjunction, ∧, is outside the parentheses and is evaluated *last,* this statement is considered a *conjunction.*

b) The comma tells us to group the statement "Dinner includes soup" with the statement "Dinner includes salad." Note that both statements are on the same

Profile in Mathematics

George Boole



The self-taught English mathematician George Boole (1815–1864) took the operations of algebra and used them to extend Aristotelian logic. He used symbols such as x and y to represent particular qualities or objects in question. For example, if x represents all butterflies, then $1 - x$ represents everything else except butterflies. If y represents the color yellow, then $(1 - x)(1 - y)$ represents everything except butterflies and things that are yellow, or yellow butterflies. This development added a computational dimension to logic that provided a basis for twentieth-century work in the field of computing.

side of the comma. The statement in symbolic form is $(p \wedge q) \vee r$. Since the disjunction, $\vee$, is outside the parentheses and is evaluated *last*, this statement is considered a *disjunction*. ■

The information provided in Example 4 is summarized below.

Statement	Symbolic representation	Type of statement
Dinner includes soup, and salad or the vegetable of the day.	$p \wedge (q \vee r)$	conjunction
Dinner includes soup and salad, or the vegetable of the day.	$(p \wedge q) \vee r$	disjunction

A negation symbol has the effect of negating only the statement that directly follows it. To negate a compound statement, we must use parentheses. When a negation symbol is placed in front of a statement in parentheses, it negates the entire statement in parentheses. The negation symbol in this case is read, "It is not true that ..." or "It is false that ..."

Example 5 *Change Symbolic Statements into Words*

Let

p: The house is for sale.
q: We can afford to buy the house.

Write the following symbolic statements in words.

a) $p \wedge \sim q$ b) $\sim p \vee \sim q$ c) $\sim(p \wedge q)$

Solution

a) The house is for sale and we cannot afford to buy the house.
b) The house is not for sale or we cannot afford to buy the house.
c) It is false that the house is for sale and we can afford to buy the house. ■

Recall that the word *but* may also be used in a conjunction. Therefore, Example 5(a) could also be written "The house is for sale, *but* we cannot afford to buy the house."

Part (b) of Example 5 is a disjunction, since it can be written $(\sim p) \vee (\sim q)$. Part (c), which is $\sim(p \wedge q)$, is a negation since the negation symbol negates the entire statement within parentheses. The similarity of these two statements is discussed in Section 3.4.

Occasionally, we come across a ***neither–nor*** statement, such as "John is neither handsome nor rich." This statement means that John is not handsome *and* John is not rich. If p represents "John is handsome" and q represents "John is rich," this statement is symbolized by $\sim p \wedge \sim q$.

If–Then Statements

The ***conditional*** is symbolized by → and is read "if–then." The statement $p \rightarrow q$ is read "If p, then q."* The conditional statement consists of two parts: the part that precedes the arrow is the ***antecedent***, and the part that follows the arrow is the ***consequent***.† In the conditional statement $p \rightarrow q$, the p is the antecedent and the q is the consequent.

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*Some books indicate that $p \rightarrow q$ may also be read "p implies q." Many higher-level mathematics books, however, indicate that $p \rightarrow q$ may be read "p implies q" only under certain conditions. Implications are discussed in Section 3.3.

†Some books refer to the antecedent as the hypothesis or premise and the consequent as the conclusion.

In the conditional statement $\sim(p \vee q) \rightarrow (p \wedge q)$, the antecedent is $\sim(p \vee q)$ and the consequent is $(p \wedge q)$. An example of a conditional statement is "If you drink your milk, then you will grow up to be healthy." A conditional symbol may be placed between any two statements even if the statements are not related.

Sometimes the word *then* in a conditional statement is not explicitly stated. For example, the statement "If you get an A, I will buy you a car" is a conditional statement because it actually means "If you get an A, then I will buy you a car."

▲ *Zoe* by Beth Anderson

Example 6 *Write Conditional Statements*

Let

p: The portrait is a pastel.
q: The portrait is by Beth Anderson.

Write the following statements symbolically.

a) If the portrait is a pastel, then the portrait is by Beth Anderson.
b) If the portrait is by Beth Anderson, then the portrait is not a pastel.
c) It is false that if the portrait is by Beth Anderson then the portrait is a pastel.

Solution

a) $p \rightarrow q$ b) $q \rightarrow \sim p$ c) $\sim(q \rightarrow p)$ ■

Example 7 *Use Commas When Writing a Symbolic Statement in Words*

Let

p: Jorge is enrolled in calculus.
q: Jorge's major is criminal justice.
r: Jorge's major is engineering.

Write the following symbolic statements in words and indicate whether the statement is a negation, conjunction, disjunction, or conditional.

a) $(q \rightarrow \sim p) \vee r$ b) $q \rightarrow (\sim p \vee r)$

Solution The parentheses indicate where to place the commas in the sentences.

a) "If Jorge's major is criminal justice then Jorge is not enrolled in calculus, or Jorge's major is engineering." This statement is a disjunction because $\vee$ is outside the parentheses.
b) "If Jorge's major is criminal justice, then Jorge is not enrolled in calculus or Jorge's major is engineering." This statement is a conditional because $\rightarrow$ is outside the parentheses. ■

If and Only if Statements

The *biconditional* is symbolized by $\leftrightarrow$ and is read "if and only if." The phrase *if and only if* is sometimes abbreviated as "iff." The statement $p \leftrightarrow q$ is read "p if and only if q."

Example 8 *Write Statements Using the Biconditional*

Let

p: Alex plays goalie on the lacrosse team.
q: The Titans win the Champion's Cup.

Write the following symbolic statements in words.

a) $p \leftrightarrow q$ c) $q \leftrightarrow \sim p$ b) $\sim(p \leftrightarrow \sim q)$

Solution

a) Alex plays goalie on the lacrosse team if and only if the Titans win the Champion's Cup.

b) The Titans win the Champion's Cup if and only if Alex does not play goalie on the lacrosse team.

c) It is false that Alex plays goalie on the lacrosse team if and only if the Titans do not win the Champion's Cup. ■

You will learn later that $p \leftrightarrow q$ means the same as $(p \rightarrow q) \wedge (q \rightarrow p)$. Therefore, the statement "I will go to college if and only if I can pay the tuition" has the same logical meaning as "If I go to college then I can pay the tuition, and if I can pay the tuition then I will go to college."

A summary of the connectives discussed in this section is given in Table 3.1.

Table 3.1 Logical Connectives

Formal Name	Symbol	Read	Symbolic Form
Negation	$\sim$	"Not"	$\sim p$
Conjunction	$\wedge$	"And"	$p \wedge q$
Disjunction	$\vee$	"Or"	$p \vee q$
Conditional	$\rightarrow$	"If-then"	$p \rightarrow q$
Biconditional	$\leftrightarrow$	"If and only if"	$p \leftrightarrow q$

SECTION 3.1 Exercises

Warm Up Exercises

In exercises 1–8, fill in the blanks with an appropriate word, phrase, or symbol(s).

1. A sentence that can be judged either true or false is called a ________.

2. A statement that conveys only one idea is called a ________ statement.

3. A statement that consists of two or more simple statements is called a ________ statement.

4. Words such as *all*, *none* (or *no*), and *some* are examples of ________.

5. a) The negation is symbolized by $\sim$ and is read "________."

b) The conjunction is symbolized by $\wedge$ and is read "________."

c) The disjunction is symbolized by $\vee$ and is read "________."

d) The conditional is symbolized by $\rightarrow$ and is read "________."

e) The biconditional is symbolized by $\leftrightarrow$ and is read "________."

6. The negation of the statement *Some cars are hybrids* is: ________ cars are hybrids.

7. The negation of the statement *All golf courses are green* is: ________ golf courses are not green.

8. The negation of the statement *Some drivers are not safe* is: ________ drivers are safe.

Practice the Skills/Problem Solving

In Exercises 9–18, indicate whether the statement is a simple statement or a compound statement. If it is a compound statement, indicate whether it is a negation, conjunction, disjunction, conditional, or biconditional by using both the word and its appropriate symbol (for example, "a negation," ~).

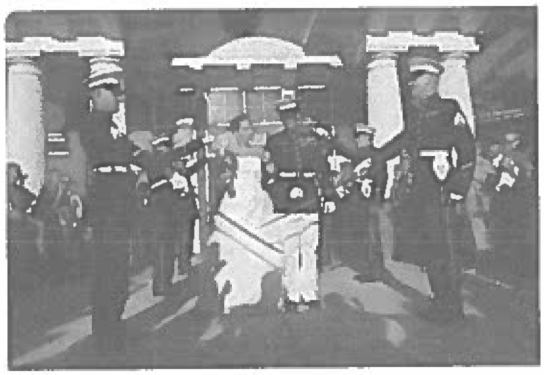
9. John Waters is scuba diving.

10. If you have a cold, then you should eat some chicken soup.

11. Time will go backwards if and only if you travel faster than the speed of light.

12. Louis Armstrong did not play the drums.

13. Bobby Glewen joined the Army and he got married.



14. The book was neither a novel nor an autobiography.

15. If Cathy Smith walks 4 miles today, then she will be sore tomorrow.

16. Inhibor Melendez will be admitted to law school if and only if he earns his bachelor's degree.

17. It is false that Jeffery Hilt is a high school teacher and a grade school teacher.

18. The hurricane did $400,000 worth of damage to DeSoto County.

In Exercises 19–28, write the negation of the statement.

19. All butterflies are insects.

20. All houses are wired using parallel circuits.

21. Some turtles do not have claws.

22. No teachers made the roster.

23. No bicycles have three wheels.

24. All horses have manes.

25. Some pedestrians are in the crosswalk.

26. Some dogs with long hair do not get cold.

27. No Holsteins are Guernseys.

28. Some vitamins contain sugar.

In Exercises 29–34, write the statement in symbolic form.

Let

p: A panther has a long tail.
q: A bobcat can purr.

29. A panther does not have a long tail.

30. A panther has a long tail and a bobcat can purr.

31. A bobcat cannot purr or a panther does not have a long tail.

32. A bobcat cannot purr if and only if a panther does not have a long tail.

33. If a panther does not have a long tail, then a bobcat cannot purr.

34. A bobcat cannot purr, but a panther has a long tail.

In Exercises 35–40, write the statement in symbolic form.

Let

p: The chili is spicy.
q: The sour cream is cold.

35. The chili is not spicy, but the sour cream is cold.

36. Neither is the chili spicy nor is the sour cream cold.

37. The sour cream is not cold if and only if the chili is spicy.

38. If the chili is spicy, then the sour cream is not cold.

39. It is false that the chili is spicy or the sour cream is cold.

40. It is false that if the sour cream is not cold then the chili is spicy.

In Exercises 41–50, write the compound statement in words.

Let

p: Joe has an iPad.
q: Brie has a MacBook.

41. $\sim q$
42. $\sim p$
43. $p \wedge q$
44. $q \vee p$
45. $\sim p \rightarrow q$
46. $\sim p \leftrightarrow \sim q$
47. $\sim p \vee \sim q$
48. $\sim(q \vee p)$
49. $\sim(p \wedge q)$
50. $\sim p \wedge \sim q$

In Exercises 51–60, write the statements in symbolic form.

Let

p: The temperature is 90°.
q: The air conditioner is working.
r: The apartment is hot.

51. The temperature is 90° and the air conditioner is not working, and the apartment is hot.

52. The temperature is not 90° and the air conditioner is working, but the apartment is hot.

53. The temperature is 90° and the air conditioner is working, or the apartment is hot.

54. If the apartment is hot and the air conditioner is working, then the temperature is 90°.

55. If the temperature is 90°, then the air conditioner is working or the apartment is not hot.

56. The temperature is not 90° if and only if the air conditioner is not working, or the apartment is not hot.

57. The apartment is hot if and only if the air conditioner is working, and the temperature is 90°.

58. It is false that if the apartment is hot then the air conditioner is not working.

59. If the air conditioner is working, then the temperature is 90° if and only if the apartment is hot.

60. The apartment is hot or the air conditioner is not working, if and only if the temperature is 90°.

In Exercises 61–70, write each symbolic statement in words.

Let

p: The water is 70°.
q: The sun is shining.
r: We go swimming.

61. $(p \vee q) \wedge \sim r$
62. $(p \wedge q) \vee r$
63. $\sim p \wedge (q \vee r)$
64. $(q \rightarrow p) \vee r$
65. $\sim r \rightarrow (q \wedge p)$
66. $(q \wedge r) \rightarrow p$
67. $(q \rightarrow r) \wedge p$
68. $\sim p \rightarrow (q \vee r)$
69. $(q \leftrightarrow p) \wedge r$
70. $q \rightarrow (p \leftrightarrow r)$

Dinner Menu *In Exercises 71–74 on page 104, use the following information to arrive at your answers. Many restaurant dinner menus include statements such as the following. All dinners are served with a choice of: Soup or Salad, and Potatoes or Pasta, and Carrots or Peas. Which of the following selections are permissible? If a selection is not permissible, explain why. See the discussion of the exclusive* or *on page 95.*

Welcome to
ANTONIO'S RESTAURANTE
"All dinners are served
with a choice of:
soup or salad, potatoes
or pasta, and carrots or peas"

71. Soup, salad, and peas

72. Salad, pasta, and carrots

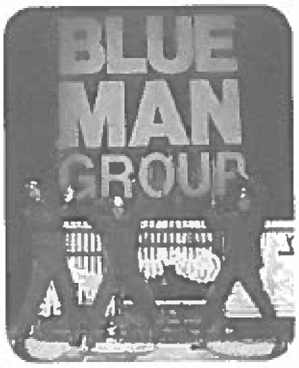
73. Soup, potatoes, pasta, and peas

74. Soup, pasta, and potatoes

In Exercises 75–83, (a) select letters to represent the simple statements and write each statement symbolically by using parentheses and (b) indicate whether the statement is a negation, conjunction, disjunction, conditional, or biconditional.

75. I bought the watch in Tijuana and I did not pay $100.

76. If the conference is in Las Vegas, then we can see Blue Man Group or we can play poker.



▲ *Blue Man Group*

77. It is false that if your speed is below the speed limit then you will not get pulled over.

78. If dinner is ready then we can eat, or we cannot go to the restaurant.

79. If the food has fiber or the food has vitamins, then you will be healthy.

80. If Corliss is teaching then Faye is in the math lab, if and only if it is not a weekend.

81. You may take this course, if and only if you did not fail the previous course or you passed the placement test.

82. If the car has gas and the battery is charged, then the car will start.

83. The classroom is empty if and only if it is the weekend, or it is 7 A.M.

Challenge Problems/Group Activities

84. ***An Ancient Question*** If Zeus could do anything, could he build a wall that he could not jump over? Explain your answer.

85. a) Make up three simple statements and label them p, q, and r. Then write compound statements to represent $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$.

b) Do you think that the statements for $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$ mean the same thing? Explain.

Recreational Mathematics

86. ***Sudoku*** Refer to the *Recreational Mathematics* on page 96. Complete the following Sudoku puzzle.

		8			3			
	9			6	1			4
	5	1	9			8	6	3
		6				4	8	
5								1
	2	4				5		
4	3	5			8	1	9	
2			5	1			7	
			2			6		

Internet/Research Activities

87. ***Legal Documents*** Obtain a legal document such as a will or rental agreement and copy one page of the document. Circle every connective used. Then list the number of times each connective appeared. Be sure to include conditional statements from which the word *then* was omitted from the sentence. Give the page and your listing to your instructor.

88. Write a report on the life and accomplishments of George Boole, who was an important contributor to the development of logic. In your report, indicate how his work eventually led to the development of the computer. References include encyclopedias, history of mathematics books, and the Internet.

SECTION 3.2

Truth Tables for Negation, Conjunction, and Disjunction

▲ *Under what conditions is the statement Subway sells the most sandwiches or Taco Bell sells the most burritos true?*

Consider the following statement: Subway sells the most sandwiches or Taco Bell sells the most burritos. Under what conditions can the statement be considered true? Under what conditions can the statement be considered false? In this section, we will introduce a tool used to help us analyze such statements.

Why ***This is Important*** Understanding when logical statements are true is essential to many different applications in everyday life.

A *truth table* is a device used to determine when a compound statement is true or false. Five basic truth tables are used in constructing other truth tables. Three are discussed in this section (Tables 3.2, 3.4, and 3.7), and two are discussed in the next section. Section 3.5 uses truth tables in determining whether a logical argument is valid or invalid.

Negation

Table 3.2 Negation

	p	$\sim p$
Case 1	T	F
Case 2	F	T

The first truth table is for *negation*. If p is a true statement, then the negation of p, "not p," is a false statement. If p is a false statement, then "not p" is a true statement. For example, if the statement "The shirt is blue" is true, then the statement "The shirt is not blue" is false. These relationships are summarized in Table 3.2. For a simple statement, there are exactly two true–false cases, as shown.

Table 3.3

	p	q
Case 1	T	T
Case 2	T	F
Case 3	F	T
Case 4	F	F

If a compound statement consists of two simple statements p and q, there are four possible cases, as illustrated in Table 3.3. Consider the statement "The test is today and the test covers Chapter 5." The simple statement "The test is today" has two possible truth values, true or false. The simple statement "The test covers Chapter 5" also has two truth values, true or false. Thus, for these two simple statements there are four distinct possible true–false arrangements. Whenever we construct a truth table for a compound statement that consists of two simple statements, we begin by listing the four true–false cases shown in Table 3.3.

Conjunction

To illustrate the conjunction, consider the following situation. You have recently purchased a new house. To decorate it, you ordered a new carpet and new furniture from the same store. You explain to the salesperson that the carpet must be delivered before the furniture. He promises that the carpet will be delivered on Thursday and that the furniture will be delivered on Friday.

Table 3.4 Conjunction

	p	q	$p \wedge q$
Case 1	T	T	T
Case 2	T	F	F
Case 3	F	T	F
Case 4	F	F	F

To help determine whether the salesperson kept his promise, we assign letters to each simple statement. Let p be "The carpet will be delivered on Thursday" and q be "The furniture will be delivered on Friday." The salesperson's statement written in symbolic form is $p \wedge q$. There are four possible true–false situations to be considered (Table 3.4).

CASE 1: p is true and q is true. The carpet is delivered on Thursday and the furniture is delivered on Friday. The salesperson has kept his promise and the compound statement is true. Thus, we put a T in the $p \wedge q$ column.

CASE 2: p is true and q is false. The carpet is delivered on Thursday but the furniture is not delivered on Friday. Since the furniture was not delivered as promised, the compound statement is false. Thus, we put an F in the $p \wedge q$ column.

CASE 3: p is false and q is true. The carpet is not delivered on Thursday but the furniture is delivered on Friday. Since the carpet was not delivered on Thursday as promised, the compound statement is false. Thus, we put an F in the $p \wedge q$ column.

CASE 4: p is false and q is false. The carpet is not delivered on Thursday and the furniture is not delivered on Friday. Since the carpet and furniture were not delivered as promised, the compound statement is false. Thus, we put an F in the $p \wedge q$ column.

Examining the four cases, we see that in only one case did the salesperson keep his promise: in case 1. Therefore, case 1 (T, T) is true. In cases 2, 3, and 4, the salesperson did not keep his promise and the compound statement is false. The results are summarized in Table 3.4, the truth table for the conjunction.

The **conjunction** $p \wedge q$ is true only when both p and q are true.

Disjunction

Civil Technician

Municipal program for redevelopment seeks on-site technician. **The applicant must have a two-year college degree in civil technology or five years of related experience.** Interested candidates please call 555-1234.

Consider the job description in the margin that describes several job requirements. Who qualifies for the job? To help analyze the statement, translate it into symbolic form. Let p be "A requirement for the job is a two-year college degree in civil technology" and q be "A requirement for the job is five years of related experience." The statement in symbolic form is $p \vee q$. For the two simple statements, there are four distinct cases (see Table 3.5).

CASE 1: p is true and q is true. A candidate has a two-year college degree in civil technology and five years of related experience. The candidate has both requirements and qualifies for the job. Consider qualifying for the job as a true statement and not qualifying as a false statement. Since the candidate qualifies for the job, we put a T in the $p \vee q$ column.

CASE 2: p is true and q is false. A candidate has a two-year college degree in civil technology but does not have five years of related experience. The candidate still qualifies for the job with the two-year college degree. Thus, we put a T in the $p \vee q$ column.

CASE 3: p is false and q is true. The candidate does not have a two-year college degree in civil technology but does have five years of related experience. The candidate still qualifies for the job with the five years of related experience. Thus, we put a T in the $p \vee q$ column.

Table 3.5 Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

CASE 4: p is false and q is false. The candidate does not have a two-year college degree in civil technology and does not have five years of related experience. The candidate does not meet either of the two requirements and therefore does not qualify for the job. Thus, we put an F in the $p \vee q$ column.

In examining the four cases, we see that there is only one case in which the candidate does not qualify for the job: case 4. As this example indicates, an *or* statement will be true in every case, except when both simple statements are false. The results are summarized in Table 3.5, the truth table for the disjunction.

The **disjunction** $p \vee q$ is true when either p is true, q is true, or both p and q are true.

The disjunction $p \vee q$ is false only when p and q are both false.

Constructing Truth Tables

We will now construct additional truth tables for statements involving the negation, conjunction, and disjunction. We summarize these compound statements on page 107.

Negation, Conjunction, and Disjunction

- **Negation** $\sim p$ is read "not p." If p is true, then $\sim p$ is false; if p is false, then $\sim p$ is true. In other words, $\sim p$ will always have the *opposite* truth value of p.
- **Conjunction** $p \wedge q$ is read "p and q." $p \wedge q$ is true only when both p and q are true.
- **Disjunction** $p \vee q$ is read "p or q." $p \vee q$ is true when either p is true, q is true, or both p and q are true. In other words, $p \vee q$ is false only when both p and q are false.

We will discuss two methods for constructing truth tables. Although the two methods produce tables that will look different, the answer columns will be the same regardless of which method you use.

Example 1 *Construct a Truth Table*

Construct a truth table for $p \wedge \sim q$.

Solution Because there are two statements, p and q, construct a truth table with four cases; see Table 3.6(a). Then write the truth values under the p in the compound statement and label this column 1, as in Table 3.6(b). Copy these truth values directly from the p column on the left. Write the corresponding truth values under the q in the compound statement and call this column 2, as in Table 3.6(c). Copy the truth values for column 2 directly from the q column on the left. Now find the truth values of $\sim q$ by negating the truth values in column 2 and call this column 3, as in Table 3.6(d). Use the conjunction table, Table 3.4, and the entries in the columns labeled 1 and 3 to complete the column labeled 4, as in Table 3.6(e). The results in column 4 are obtained as follows:

Table 3.6

(a)

	p	q	$p \wedge \sim q$
Case 1	T	T	
Case 2	T	F	
Case 3	F	T	
Case 4	F	F	

(b)

p	q	$p \wedge \sim q$
T	T	T
T	F	T
F	T	F
F	F	F
		1

(c)

p	q	p	$\wedge$	$\sim$	q
T	T	T			T
T	F	T			F
F	T	F			T
F	F	F			F
		1			2

(d)

p	q	p	$\wedge$	$\sim$	q
T	T	T		F	T
T	F	T		T	F
F	T	F		F	T
F	F	F		T	F
		1		3	2

(e)

p	q	p	$\wedge$	$\sim$	q
T	T	T	F	F	T
T	F	T	T	T	F
F	T	F	F	F	T
F	F	F	F	T	F
		1	4	3	2

Row 1: T ∧ F is F. Row 2: T ∧ T is T.
Row 3: F ∧ F is F. Row 4: F ∧ T is F.

The answer is always the last column completed. The columns labeled 1, 2, and 3 are only aids in arriving at the answer labeled column 4. ■

The statement $p \wedge \sim q$ in Example 1 actually means $p \wedge (\sim q)$. In the future, instead of listing a column for q and a separate column for its negation, we will make one column for $\sim q$, which will have the opposite values of those in the q column on the left. Similarly, when we evaluate $\sim p$, we will use the opposite values of those in the p column on the left. This procedure is illustrated in Example 2.

In Example 1, we spoke about *cases* and also *columns*. Consider Table 3.6(e) on page 107. This table has four cases indicated by the four different rows of the two left-hand (unnumbered) columns. The four *cases* are TT, TF, FT, and FF. In every truth table with two letters, we list the four cases (the first two columns) first. Then we complete the remaining columns in the truth table. In Table 3.6(e), after completing the two left-hand columns, we complete the remaining columns in the order indicated by the numbers below the columns. We will continue to place numbers below the columns to show the order in which the columns are completed.

In discussion of the truth table in Example 2, and all following truth tables, if we say column 1, it means the column labeled 1. Column 2 will mean the column labeled 2, and so on.

TIMELY TIP

When constructing truth tables it is very important to keep your entries in neat columns and rows. If you are using lined paper, put only one row of the table on each line. If you are not using lined paper, using a straightedge may help you correctly enter the information into the truth table's rows and columns.

Example 2 *Construct and Interpret a Truth Table*

a) Construct a truth table for the following statement: Jo is not an artist and Jo is not a musician.

b) Under which conditions will the compound statement be true?

c) Suppose "Jo is an artist" is a false statement and "Jo is a musician" is a true statement. Is the compound statement given in part (a) true or false?

Solution

a) First write the simple statements in symbolic form by using simple nonnegated statements.

Let

p: Jo is an artist.
q: Jo is a musician.

Therefore, the compound statement may be written $\sim p \wedge \sim q$. Now construct a truth table with four cases, as shown in Table 3.7.

Table 3.7

p	q	$\sim p$	$\wedge$	$\sim q$
T	T	F	F	F
T	F	F	F	T
F	T	T	Ⓕ	F
F	F	T	Ⓣ	T
		1	3	2

Fill in the column labeled 1 by negating the truth values under p on the far left. Fill in the column labeled 2 by negating the values under q in the second column from the left. Fill in the column labeled 3 by using the columns labeled 1 and 2 and the definition of conjunction.

In the first row, to determine the entry for column 3, we use false for $\sim p$ and false for $\sim q$. Since false $\wedge$ false is false (see case 4 of Table 3.4 on page 105), we place an F in column 3, row 1. In the second row, we use false for $\sim p$ and true for $\sim q$. Since false $\wedge$ true is false (see case 3 of Table 3.4), we place an F in column 3, row 2. In the third row, we use true for $\sim p$ and false for $\sim q$. Since true $\wedge$ false is false (see case 2 of Table 3.4), we place an F in column 3, row 3. In the fourth row, we use true for $\sim p$ and true for $\sim q$. Since true $\wedge$ true is true (see case 1 of Table 3.4), we place a T in column 3, row 4.

b) The compound statement in part (a) will be true only in case 4 (circled in blue) when both simple statements, p and q, are false, that is, when Jo is not an artist and Jo is not a musician.

c) We are told that p, "Jo is an artist," is a false statement and that q, "Jo is a musician," is a true statement. From the truth table (Table 3.7), we can determine that when p is false and q is true, the compound statement, case 3 (circled in red), is false. ■

Table 3.8

p	q	$\sim$	$(\sim q$	$\vee$	$p)$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	F	T	T	F
		4	1	3	2

Example 3 *Truth Table with a Negation*

Construct a truth table for $\sim(\sim q \vee p)$.

Solution First construct the standard truth table listing the four cases. Then work within parentheses. The order to be followed is indicated by the numbers below the columns (see Table 3.8). Under $\sim q$, column 1, write the negation of the q column. Then, in column 2, copy the values from the p column. Next, complete the *or* column, column 3, using columns 1 and 2 and the truth table for the disjunction (see Table 3.5 on page 106). The *or* column is false only when both statements are false, as in case 3. Finally, negate the values in the *or* column, column 3, and place these negated values in column 4. By examining the truth table you can see that the compound statement $\sim(\sim q \vee p)$ is true only in case 3, that is, when p is false and q is true. ■

PROCEDURE CONSTRUCTING TRUTH TABLES

1. Study the compound statement and determine whether it is a negation, conjunction, disjunction, conditional, or biconditional statement, as was done in Section 3.1. The answer to the truth table will appear under $\sim$ if the statement is a negation, under $\wedge$ if the statement is a conjunction, under $\vee$ if the statement is a disjunction, under $\rightarrow$ if the statement is a conditional, and under $\leftrightarrow$ if the statement is a biconditional.
2. Complete the columns under the simple statements, p, q, r, and their negations, $\sim p$, $\sim q$, $\sim r$, within parentheses, if present. If there are nested parentheses (one pair of parentheses within another pair), work with the innermost pair first.
3. Complete the column under the connective within the parentheses, if present. You will use the truth values of the connective in determining the final answer in step 5.
4. Complete the column under any remaining statements and their negations.
5. Complete the column under any remaining connectives. Recall that the answer will appear under the column determined in step 1. If the statement is a conjunction, disjunction, conditional, or biconditional, you will obtain the truth values for the connective by using the last column completed on the left side and on the right side of the connective. If the statement is a negation, you will obtain the truth values by negating the truth values of the last column completed within the grouping symbols on the right side of the negation. Be sure to circle or highlight your answer column or number the columns in the order they were completed.

Table 3.9

p	q	$(\sim p$	$\vee$	$q)$	$\wedge$	$\sim p$
T	T	F	T	T	F	F
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	F	T	T	F	T	T
		1	3	2	5	4

Example 4 *Use the General Procedure to Construct a Truth Table*

Construct a truth table for the statement $(\sim p \vee q) \wedge \sim p$.

Solution We will follow the general procedure outlined in the box. This statement is a conjunction, so the answer will be under the conjunction symbol. Complete columns under $\sim p$ and q within the parentheses and call these columns 1 and 2, respectively (see Table 3.9). Complete the column under the disjunction, $\vee$, using the truth values in columns 1 and 2, and call this column 3. Next complete the column under $\sim p$, and call this column 4. The answer, column 5, is determined from the definition of the conjunction and the truth values in column 3, the last column completed on the left side of the conjunction, and column 4. ■

So far, all the truth tables we have constructed have contained at most two simple statements. Now we will explain how to construct a truth table that consists of three simple statements, such as $(p \wedge q) \wedge r$. When a compound statement consists of three simple statements, there are eight different true–false possibilities, as illustrated in Table 3.10. To begin such a truth table, write four Ts and four Fs in the column under p. Under the second statement, q, pairs of Ts alternate with pairs of Fs. Under the third statement, r, T alternates with F. This technique is not the only way of listing the cases, but it ensures that each case is unique and that no cases are omitted.

Table 3.10

	p	q	r
Case 1	T	T	T
Case 2	T	T	F
Case 3	T	F	T
Case 4	T	F	F
Case 5	F	T	T
Case 6	F	T	F
Case 7	F	F	T
Case 8	F	F	F

Example 5 *Construct a Truth Table with Eight Cases*

a) Construct a truth table for the statement "Santana is home and he is not at his desk, or he is sleeping."

b) Suppose that "Santana is home" is a false statement, that "Santana is at his desk" is a true statement, and that "Santana is sleeping" is a true statement. Is the compound statement in part (a) true or false?

Solution

a) First we will translate the statement into symbolic form.

Let

p: Santana is home.
q: Santana is at his desk.
r: Santana is sleeping.

In symbolic form, the statement is $(p \wedge \sim q) \vee r$.

Since the statement is composed of three simple statements, there are eight cases. Begin by listing the eight cases in the three left-hand columns; see Table 3.11. By examining the statement, you can see that it is a disjunction. Therefore, the answer will be in the $\vee$ column. Fill out the truth table by working in parentheses first. Place values under p, column 1, and $\sim q$, column 2. Then find the conjunctions of columns 1 and 2 to obtain column 3. Place the values of r in column 4. To obtain the answer, column 5, use columns 3 and 4 and the information for the disjunction contained in Table 3.5 on page 106.

Table 3.11

p	q	r	$(p$	$\wedge$	$\sim q)$	$\vee$	r
T	T	T	T	F	F	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	F
F	T	T	F	F	F	T	T
F	T	F	F	F	F	F	F
F	F	T	F	F	T	T	T
F	F	F	F	F	T	F	F
			1	3	2	5	4

b) We are given the following:

p: Santana is home—false.
q: Santana is at his desk—true.
r: Santana is sleeping—true.

We need to find the truth value of the following case: false, true, true. In case 5 of the truth table, p, q, and r are F, T, and T, respectively. Therefore, under these conditions, the original compound statement is true (as circled in the table). ■

We have learned that a truth table with one simple statement has two cases, a truth table with two simple statements has four cases, and a truth table with three simple statements has eight cases. In general, *the number of distinct cases in a truth table with n distinct simple statements is* 2^n. The compound statement $(p \vee q) \vee (r \wedge \sim s)$ has four simple statements, p, q, r, s. Thus, a truth table for this compound statement would have 2^4, or 16, distinct cases.

When we construct a truth table, we determine the truth values of a compound statement for every possible case. If we want to find the truth value of the compound statement for any specific case when we know the truth values of the simple statements, we do not have to develop the entire table. For example, to determine the truth value for the statement

$$2 + 3 = 5 \qquad \text{and} \qquad 1 + 1 = 3$$

we let p be $2 + 3 = 5$ and q be $1 + 1 = 3$. Now we can write the compound statement as $p \wedge q$. We know that p is a true statement and q is a false statement. Thus, we can substitute T for p and F for q and evaluate the statement:

$$\begin{array}{c} p \wedge q \\ \mathrm{T} \wedge \mathrm{F} \\ \mathrm{F} \end{array}$$

Therefore, the compound statement $2 + 3 = 5$ and $1 + 1 = 3$ is a false statement.

Alternate Method for Constructing Truth Tables

We now present an alternate method for constructing truth tables. We will use the alternate method to construct truth tables for the same statements we analyzed in Examples 1, 2, and 3.

Example 6 *Use the Alternate Method to Construct a Truth Table*

Construct a truth table for $p \wedge \sim q$.

Solution We begin by constructing the first two columns of a truth table with four cases, as shown in Table 3.12 (a). We will add additional columns to Table 3.12(a) to develop our answer column. Since we wish to find the truth table for the compound statement $p \wedge \sim q$, we need to be able to compare the truth values for p with the truth values for $\sim q$. Table 3.12(a) already has a column showing the truth values for p. We next add a column showing the truth values for $\sim q$, as shown in Table 3.12(b). Recall that the values of $\sim q$ are the opposite of those for q.

Finally, we add the answer column for the compound statement $p \wedge \sim q$, as shown as shown in Table 3.12(c). To determine the truth values for the $p \wedge \sim q$ column, use the p column and the $\sim q$ column, and the conjunction table, Table 3.4,

Table 3.12

(a)

	p	q
Case 1	T	T
Case 2	T	F
Case 3	F	T
Case 4	F	F

(b)

p	q	$\sim q$
T	T	F
T	F	T
F	T	F
F	F	T

(c)

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Use these columns to determine the answer column on page 105. Note that the answer column of Table 3.12(c) is the same as the answer column of Table 3.6(e) on page 107. ■

Example 7 *Use the Alternate Method to Construct a Truth Table*

Construct a truth table for $\sim p \wedge \sim q$.

Solution Begin by constructing a truth table with four cases, as shown in Table 3.13(a). Since we wish to find the truth table for the compound statement $\sim p \wedge \sim q$, we will add a column for $\sim p$ and a column for $\sim q$, as shown in Table 3.13(b).

Table 3.13

(a)

p	q
T	T
T	F
F	T
F	F

(b)

p	q	$\sim p$	$\sim q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

(c)

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Use these columns to determine the answer column

Finally, we add our answer column for the compound statement $\sim p \wedge \sim q$. To determine the truth values for the $\sim p \wedge \sim q$ column, use the $\sim p$ column and the $\sim q$ column, and the conjunction table, Table 3.4, on page 105. Note that the answer column of Table 3.13(c) is the same as the answer column of Table 3.7 on page 108. ■

Example 8 *Use the Alternate Method to Construct a Truth Table*

Construct a truth table for $\sim(\sim q \vee p)$.

Solution Begin by constructing a truth table with four cases, as shown in Table 3.14(a). To complete the truth table, we will work within parentheses first. Thus, we next add a column for $\sim q$, as shown in Table 3.14(b). We then will construct a column for the expression within parentheses $\sim q \vee p$ by using the $\sim q$ column and the p column, and the disjunction table, Table 3.5, on page 106.

Table 3.14

(a)

p	q
T	T
T	F
F	T
F	F

(b)

p	q	$\sim q$
T	T	F
T	F	T
F	T	F
F	F	T

(c)

p	q	$\sim q$	$\sim q \vee p$
T	T	F	T
T	F	T	T
F	T	F	F
T	F	T	T

Use these columns to determine the $\sim q \vee p$ column

Finally, we add the answer column for the compound statement, $\sim(\sim q \vee p)$, as shown in Table 3.15. To determine the truth values for the $\sim(\sim q \vee p)$ column, take the opposite values of those shown in the $\sim q \vee p$ column. Note that the answer column of Table 3.15 is the same as the answer column of Table 3.8 on page 109. ■

Table 3.15

p	q	$\sim q$	$\sim q \vee p$	$\sim(\sim q \vee p)$
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

Take the opposite of this column to get the answer column

We have demonstrated two methods for constructing truth tables. Unless your instructor indicates otherwise, you may use either method. Although both methods will always lead to the correct answer, the alternate method gets more cumbersome and takes up more space as we construct truth tables with more statements. *In the remainder of this chapter, we will demonstrate the construction of truth tables using only the first method.*

Determine Truth Values Without Constructing a Truth Table

In the remaining examples in this section, we will determine the truth values of compound statements without constructing a truth table.

Example 9 *Determine the Truth Value of a Compound Statement*

Determine the truth value for each simple statement. Then, using these truth values, determine the truth value of the compound statement.

a) 15 is less than or equal to 9.

b) George Washington was the first U.S. president or Abraham Lincoln was the second U.S. president, but there has not been a U.S. president born in Antarctica.

Solution

a) Let

p: 15 is less than 9.
q: 15 is equal to 9.

The statement "15 is less than or equal to 9" means that 15 is less than 9 or 15 is equal to 9. The compound statement can be expressed as $p \vee q$. We know that both p and q are false statements, since 15 is greater than 9, so we substitute F for p and F for q and evaluate the statement:

$$\begin{gathered} p \vee q \\ \mathrm{F} \vee \mathrm{F} \\ \mathrm{F} \end{gathered}$$

Therefore, the compound statement "15 is less than or equal to 9" is a false statement.

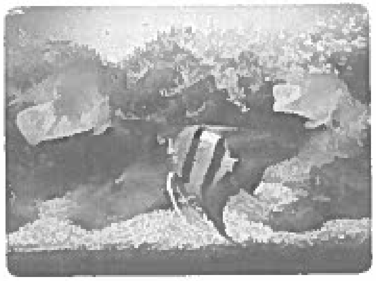
b) Let

p: George Washington was the first U.S. president.
q: Abraham Lincoln was the second U.S. president.
r: There has been a U.S. president who was born in Antarctica.

The compound statement can be written in symbolic form as $(p \vee q) \wedge \sim r$. Recall that *but* is used to express a conjunction. We know that p is a true statement and that q is a false statement. We also know that r is a false statement since all U.S. presidents must be born in the United States. Thus, since r is a false statement, the negation, $\sim r$, is a true statement. So we will substitute T for p, F for q, and T for $\sim r$ and then evaluate the statement:

$$\begin{gathered} (p \vee q) \wedge \sim r \\ (\mathrm{T} \vee \mathrm{F}) \wedge \mathrm{T} \\ \mathrm{T} \wedge \mathrm{T} \\ \mathrm{T} \end{gathered}$$

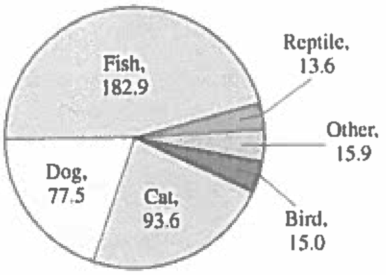
Therefore, the original compound statement is a true statement. ■



Example 10 *Pet Ownership in the United States*

The number of pets owned in the United States in 2009 is shown in Figure 3.1. Use this graph to determine the truth value of the following statement: There are more dogs owned than cats and there are fewer reptiles owned than birds, or the most numerous pets owned are not fish.

Pets Owned in the United States (millions)



Source: American Pet Products Manufacturing Association

Figure 3.1

Solution Let

p: There are more dogs owned than cats.
q: There are fewer reptiles owned than birds.
r: The most numerous pets owned are fish.

The given compound statement can be written in symbolic form as $(p \wedge q) \vee \sim r$. From Fig. 3.1, we see that statement p is false: There are actually more cats owned than dogs. We also see that statement q is true: There are fewer reptiles owned than birds. We also see that statement r is true: The most numerous pets owned are fish. Since r is true, its negation, $\sim r$, is false. Therefore, we substitute F for p, T for q, and F for $\sim r$, which gives

$$\begin{gathered}(p \wedge q) \vee \sim r \\ (\mathrm{F} \wedge \mathrm{T}) \vee \mathrm{F} \\ \mathrm{F} \vee \mathrm{F} \\ \mathrm{F}\end{gathered}$$

Thus, the original compound statement is a false statement. ■

SECTION 3.2 Exercises

Warm Up Exercises

In Exercises 1–4, fill in the blanks with an appropriate word, phrase, or symbol(s).

1. The negation $\sim p$ will always have the ________ truth value of p.

2. The conjunction $p \wedge q$ is true only when both p and q are ________.

3. The disjunction $p \vee q$ is false only when both p and q are ________.

4. A truth table for a compound statement with
   a) one distinct simple statement will have ________ cases.
   b) two distinct simple statements will have ________ cases.
   c) three distinct simple statements will have ________ cases.

Practice the Skills/Problem Solving

In Exercises 5–18, construct a truth table for the statement.

5. $p \wedge \sim p$

6. $p \vee \sim p$

7. $q \vee \sim p$

8. $p \wedge \sim q$

9. $\sim p \vee \sim q$

10. $\sim(p \vee \sim q)$

11. $\sim(p \wedge \sim q)$

12. $\sim(\sim p \wedge \sim q)$

13. $\sim q \vee (p \wedge r)$

14. $(p \vee \sim q) \wedge r$

15. $r \vee (p \wedge \sim q)$

16. $(r \wedge q) \wedge \sim p$

17. $(r \vee \sim p) \wedge \sim q$

18. $\sim p \wedge (q \vee r)$

In Exercises 19–26, write the statement in symbolic form and construct a truth table.

19. Train recorded *Hey, Soul Sister* and The Black Eyed Peas recorded *Where Is the Love?*

▲ *The Black Eyed Peas*

20. We can eat at McDonald's, but we cannot eat breakfast.

21. I have worked all week, but I have not been paid.

22. It is false that Robert A. Farinelli is the president or that Pauline Chow is the treasurer.

23. It is false that Jasper Adams is a tutor and Mark Russo is a secretary.

24. Mike made pizza and Dennis made a chef salad, but Gil burned the lemon squares.

25. The copier is out of toner, or the lens is dirty or the corona wires are broken.

26. I am hungry, and I want to eat a healthy lunch and I want to eat in a hurry.

In Exercises 27–36, determine the truth value of the statement if

a) p is true, q is false, and r is true.

b) p is false, q is true, and r is true.

27. $(p \vee q) \wedge \sim r$

28. $p \vee (q \wedge \sim r)$

29. $(\sim p \wedge \sim q) \vee \sim r$

30. $\sim p \wedge (\sim q \vee \sim r)$

31. $(p \vee \sim q) \wedge \sim(p \wedge \sim r)$

32. $(p \wedge \sim q) \vee r$

33. $(\sim r \wedge p) \vee q$

34. $\sim q \vee (r \wedge p)$

35. $(\sim p \vee \sim q) \vee (\sim r \vee q)$

36. $(\sim r \wedge \sim q) \wedge (\sim r \vee \sim p)$

In Exercises 37–44, determine the truth value for each simple statement. Then use these truth values to determine the truth value of the compound statement. (You may have to use a reference source such as the Internet or an encyclopedia.)

37. $8 + 7 = 20 - 5$ and $63 \div 7 = 3 \cdot 3$

38. $0 < -3$ or $5 \geq 10$

39. Florida is in Canada or Texas borders Mexico.

40. Washington, DC is west of the Mississippi River and Virginia is an island in the Pacific Ocean.

41. Quentin Tarantino is a movie director and Zac Efron is an actor, but Scarlett Johansson is not an actress.

▲ *Scarlett Johansson*

42. Quebec is in Texas or Toronto is in California, and Cedar Rapids is in Iowa.

43. Iraq is in Africa or Iran is in South America, and Syria is in the Middle East.

44. Holstein is a breed of cattle and collie is a breed of dogs, or beagle is not a breed of cats.

Number of Movies Produced *In Exercises 45–48, use the table to determine the truth value of each simple statement. Then determine the value of the compound statement.*

Countries with the Highest Number of Feature Films Produced in 2008

Country	Number of Films
India	1325
United States	520
Japan	418
China	406
France	240

Source: Screen Digest Magazine

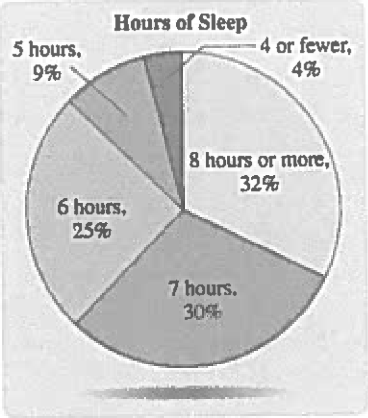
45. India produced more feature films than the United States and China produced more feature films than Japan.

46. India produced more than twice as many feature films as the United States and India produced more feature films than Japan, China, and France combined.

47. The United States produced more feature films than Japan, but Japan produced more feature films than China.

48. The United States produced more than twice as many feature films as France or Japan produced more feature films than India.

Sleep Time *In Exercises 49–52, use the graph, which shows the number of hours Americans sleep, to determine the truth value of each simple statement. Then determine the truth value of the compound statement.*



49. It is false that 30% of Americans get 6 hours of sleep each night and 9% get 5 hours of sleep each night.

50. Twenty-five percent of Americans get 6 hours of sleep each night, and 30% get 7 hours of sleep each night or 9% do not get 5 hours of sleep each night.

51. Thirteen percent of Americans get 5 or fewer hours of sleep each night or 32% get 6 or more hours of sleep each night, and 30% get 8 or more hours of sleep each night.

52. Over one-half of all Americans get 7 or fewer hours of sleep each night, and over one-quarter get 6 or fewer hours of sleep each night.

In Exercises 53–56, let

p: Tanisha owns a convertible.
q: Joan owns a Volvo.

Translate each statement into symbols. Then construct a truth table for each compound statement and indicate under what conditions the compound statement is true.

53. Tanisha owns a convertible and Joan does not own a Volvo.

54. Tanisha does not own a convertible, but Joan owns a Volvo.

55. Tanisha owns a convertible or Joan does not own a Volvo.

56. Tanisha does not own a convertible or Joan does not own a Volvo.

In Exercises 57–60 on page 117, let

p: The house is owned by an engineer.
q: The heat is solar generated.
r: The car is run by electric power.

Translate each statement into symbols. Then construct a truth table for each and indicate under what conditions the compound statement is true.

57. The car is run by electric power or the heat is solar generated, but the house is owned by an engineer.

58. The house is owned by an engineer and the heat is solar generated, or the car is run by electric power.

59. The heat is solar generated, or the house is owned by an engineer and the car is not run by electric power.

60. The house is not owned by an engineer, and the car is not run by electric power and the heat is solar generated.

Obtaining a Loan *In Exercises 61 and 62, read the requirements and each applicant's qualifications for obtaining a loan.*

a) Identify which of the applicants would qualify for the loan.

b) For the applicants who do not qualify for the loan, explain why.

61. To qualify for a loan of $40,000, an applicant must have a gross income of $28,000 if single, $46,000 combined income if married, and assets of at least $6,000.

Mrs. Rusinek, married with three children, earns $42,000. Mr. Rusinek does not have an income. The Rusineks have assets of $42,000.

Mr. Duncan is not married, works in sales, and earns $31,000. He has assets of $9000.

Mrs. Tuttle and her husband have total assets of $43,000. One earns $35,000, and the other earns $23,500.

62. To qualify for a loan of $45,000, an applicant must have a gross income of $30,000 if single, $50,000 combined income if married, and assets of at least $10,000.

Mr. Argento, married with two children, earns $37,000. Mrs. Argento earns $15,000 at a part-time job. The Argentos have assets of $25,000.

Ms. McVey, single, has assets of $19,000. She works in a store and earns $25,000.

Mr. Siewert earns $24,000 and Ms. Fox, his wife, earns $28,000. Their assets total $8000.

63. ***Airline Special Fares*** An airline advertisement states, "To get the special fare you must purchase your tickets between January 1 and February 15 and fly round trip between March 1 and April 1. You must depart on a Monday, Tuesday, or Wednesday, and return on a Tuesday, Wednesday, or Thursday, and stay over at least one Saturday."

a) Determine which of the following individuals will qualify for the special fare.

b) If the person does not qualify for the special fare, explain why.

Wing Park plans to purchase his ticket on January 15, depart on Monday, March 3, and return on Tuesday, March 18.

Gina Vela plans to purchase her ticket on February 1, depart on Wednesday, March 12, and return on Thursday, April 3.

Kara Shavo plans to purchase her ticket on February 14, depart on Tuesday, March 4, and return on Monday, March 19.

Christos Supernaw plans to purchase his ticket on January 4, depart on Monday, March 10, and return on Thursday, March 13.

Alex Chang plans to purchase his ticket on January 1, depart on Monday, March 3, and return on Monday, March 10.

▲ See Exercise 63

Problem Solving/Group Activities

In Exercises 64 and 65, construct a truth table for the symbolic statement.

64. $\sim[(\sim(p \vee q)) \vee (q \wedge r)]$

65. $[(q \wedge \sim r) \wedge (\sim p \vee \sim q)] \vee (p \vee \sim r)$

66. On page 111, we indicated that a compound statement consisting of n simple statements had 2^n distinct true–false cases.

a) How many distinct true–false cases does a truth table containing simple statements p, q, r, and s have?

b) List all possible true–false cases for a truth table containing the simple statements p, q, r, and s.

c) Use the list in part (b) to construct a truth table for $(q \wedge p) \vee (\sim r \wedge s)$.

d) Construct a truth table for $(\sim r \wedge \sim s) \wedge (\sim p \vee q)$.

67. Must $(p \wedge \sim q) \vee r$ and $(q \wedge \sim r) \vee p$ have the same number of trues in their answer columns? Explain.

Internet/Research Activities

68. Do research and write a report on each of the following.

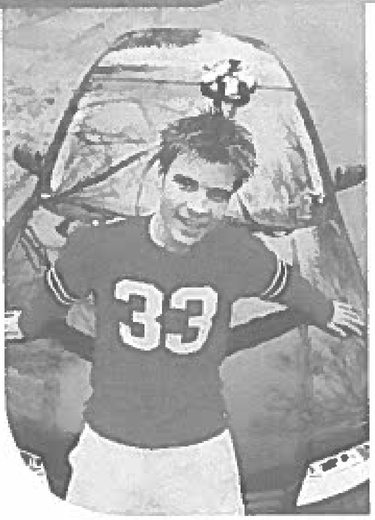
a) The relationship between *negation* in logic and *complement* in set theory.

b) The relationship between *conjunction* in logic and *intersection* in set theory.

c) The relationship between *disjunction* in logic and *union* in set theory.

SECTION 3.3

Truth Tables for the Conditional and Biconditional



▲ *Under what conditions is the statement* If you get an A, then I will buy you a car *true?*

Suppose I said to you, "If you get an A, then I will buy you a car." As we discussed in Section 3.1, this statement is called a *conditional* statement. In this section, we will discuss under what conditions a conditional statement is true and under what conditions a conditional statement is false.

Why *This Is Important* Understanding when conditional statements are true is essential to understanding logic and real-life documents such as wills, trusts, and contracts.

Conditional

In Section 3.1, we mentioned that the statement preceding the conditional symbol is called the *antecedent* and that the statement following the conditional symbol is called the *consequent*. For example, consider $(p \vee q) \rightarrow [\sim(q \wedge r)]$. In this statement, $(p \vee q)$ is the antecedent and $[\sim(q \wedge r)]$ is the consequent.

To develop a truth table for the conditional statement, consider the statement "If you get an A, then I will buy you a car." Assume this statement is true except when I have actually broken my promise to you.

Let

p: You get an A.
q: I buy you a car.

Translated into symbolic form, the statement becomes $p \rightarrow q$. Let's examine the four cases shown in Table 3.16.

Table 3.16 Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

CASE 1: (T, T) You get an A, and I buy a car for you. I have met my commitment, and the statement is true.

CASE 2: (T, F) You get an A, and I do not buy a car for you. I have broken my promise, and the statement is false.

What happens if you don't get an A? If you don't get an A, I no longer have a commitment to you, and therefore I cannot break my promise.

CASE 3: (F, T) You do not get an A, and I buy you a car. I have not broken my promise, and therefore the statement is true.

CASE 4: (F, F) You do not get an A, and I don't buy you a car. I have not broken my promise, and therefore the statement is true.

The conditional statement is false when the antecedent is true and the consequent is false. In every other case the conditional statement is true.

The **conditional statement** $p \rightarrow q$ is true in every case except when p is a true statement and q is a false statement.

Example 1 *A Truth Table with a Conditional*

Construct a truth table for the statement $\sim p \rightarrow \sim q$.

Solution Because this statement is a conditional, the answer will lie under the →. Fill out the truth table by placing the appropriate truth values under $\sim p$, column 1, and under $\sim q$, column 2 (see Table 3.17). Then, using the information given in the truth table for the conditional (Table 3.16 on page 118) and the truth values in columns 1 and 2, determine the solution, column 3. In row 1, the antecedent, $\sim p$, is false and the consequent, $\sim q$, is also false. Row 1 is F → F, which according to row 4 of Table 3.16, is T. Likewise, row 2 of Table 3.17 is F → T, which is T. Row 3 is T → F, which is F. Row 4 is T → T, which is T. ■

Table 3.17

p	q	$\sim p$	$\rightarrow$	$\sim q$
T	T	F	T	F
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T
		1	3	2

Example 2 *A Conditional Truth Table with Three Simple Statements*

Construct a truth table for the statement $p \rightarrow (\sim q \wedge r)$.

Solution Because this statement is a conditional, the answer will lie under the →. Work within the parentheses first. Place the truth values under $\sim q$, column 1, and r, column 2 (Table 3.18). Then take the conjunction of columns 1 and 2 to obtain column 3. Next, place the truth values under p in column 4. To determine the answer, column 5, use columns 3 and 4 and the information of the conditional statement given in Table 3.16. Column 4 represents the truth values of the antecedent, and column 3 represents the truth values of the consequent. Remember that the conditional is false only when the antecedent is true and the consequent is false, as in cases (rows) 1, 2, and 4 of column 5. ■

Table 3.18

p	q	r	p	$\rightarrow$	$(\sim q$	$\wedge$	$r)$
T	T	T	T	F	F	F	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	F	F	T
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	F	F
			4	5	1	3	2

Example 3 *Examining an Advertisement*

An advertisement for Perky Morning coffee makes the following claim: "If you drink Perky Morning coffee, then you will not be sluggish and you will have a great day." Translate the statement into symbolic form and construct a truth table.

Solution Let

p: You drink Perky Morning coffee.
q: You will be sluggish.
r: You will have a great day.

In symbolic form, the claim is

$$p \rightarrow (\sim q \wedge r)$$

This symbolic statement is identical to the statement in Table 3.18, and the truth tables are the same. Column 3 represents the truth values of $(\sim q \wedge r)$, which corresponds to the statement "You will not be sluggish and you will have a great day." Note that column 3 is true in cases (rows) 3 and 7. In case 3, since p is true, you drank Perky Morning coffee. In case 7, however, since p is false, you did not drink Perky Morning coffee. From this information we can conclude that it is possible for you to not be sluggish and for you to have a great day without drinking Perky Morning coffee. ■

A truth table cannot by itself determine whether a compound statement is true or false. However, a truth table does allow us to examine all possible cases for compound statements.

Biconditional

The *biconditional statement* $p \leftrightarrow q$ means that $p \rightarrow q$ and $q \rightarrow p$, or, symbolically, $(p \rightarrow q) \wedge (q \rightarrow p)$. To determine the truth table for $p \leftrightarrow q$, we will construct the truth table for $(p \rightarrow q) \wedge (q \rightarrow p)$ (Table 3.19 on page 120). Table 3.20 on page 120 shows the truth values for the biconditional statement.

Table 3.19

p	q	$(p$	$\to$	$q)$	$\wedge$	$(q$	$\to$	$p)$
T	T	T	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T
F	T	F	T	T	F	T	F	F
F	F	F	T	F	T	F	T	F
		1	3	2	7	4	6	5

Table 3.20 Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

From Table 3.20 we see that the biconditional statement is true when the antecedent and the consequent have the same truth value and false when the antecedent and consequent have different truth values.

The **biconditional statement** $p \leftrightarrow q$ is true only when p and q have the same truth value, that is, when both are true or both are false.

Example 4 *A Truth Table Using a Biconditional*

Construct a truth table for the statement $\sim p \leftrightarrow (\sim q \to r)$.

Solution Since there are three letters, there must be eight cases. The parentheses indicate that the answer must be under the biconditional, as shown in Table 3.21. Use columns 1 and 4 to obtain the answer in column 5. When columns 1 and 4 have the same truth values, place a T in column 5. When columns 1 and 4 have different truth values, place an F in column 5.

Table 3.21

p	q	r	$\sim p$	$\leftrightarrow$	$(\sim q$	$\to$	$r)$
T	T	T	F	F	F	T	T
T	T	F	F	F	F	T	F
T	F	T	F	F	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	F	T	F	F
			1	5	2	4	3

■

In Section 3.2, we showed that finding the truth value of a compound statement for a specific case does not require constructing an entire truth table. Examples 5 and 6 illustrate this technique for the conditional and the biconditional.

Example 5 *Determine the Truth Value of a Compound Statement*

Determine the truth value of the statement $(\sim p \leftrightarrow q) \to (\sim q \leftrightarrow r)$ when p is false, q is true, and r is false.

Solution Substitute the truth value for each simple statement:

$$\begin{array}{c}
(\sim p \leftrightarrow q) \rightarrow (\sim q \leftrightarrow r) \\
(\mathrm{T} \leftrightarrow \mathrm{T}) \;\rightarrow\; (\mathrm{F} \leftrightarrow \mathrm{F}) \\
\mathrm{T} \quad \rightarrow \quad \mathrm{T} \\
\mathrm{T}
\end{array}$$

For this specific case, the statement is true. ■

Example 6 *Determine the Truth Value of a Compound Statement*

Determine the truth value for each simple statement. Then use the truth values to determine the truth value of the compound statement.

a) If 15 is an even number, then 29 is an even number.

b) Vanderbilt University is in Tennessee and Wake Forest University is in Alaska, if and only if Syracuse University is in Alabama.

Solution

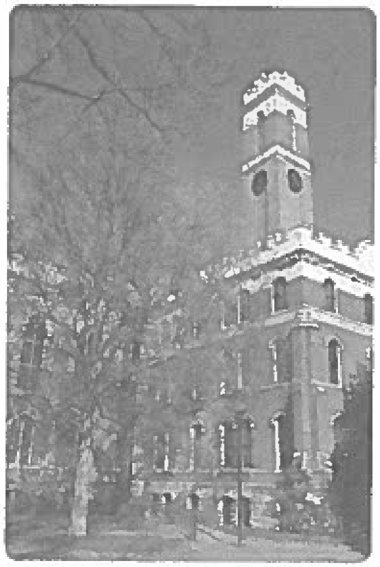
a) Let

p: 15 is an even number.
q: 29 is an even number.

Then the statement "If 15 is an even number, then 29 is an even number" can be written $p \rightarrow q$. Since 15 is not an even number, p is a false statement. Also, since 29 is not an even number, q is a false statement. We substitute F for p and F for q and evaluate the statement:

$$\begin{array}{c}
p \rightarrow q \\
\mathrm{F} \rightarrow \mathrm{F} \\
\mathrm{T}
\end{array}$$

Therefore, "If 15 is an even number, then 29 is an even number" is a true statement.



▲ *Vanderbilt University*

b) Let

p: Vanderbilt University is in Tennessee.
q: Wake Forest University is in Alaska.
r: Syracuse University is in Alabama.

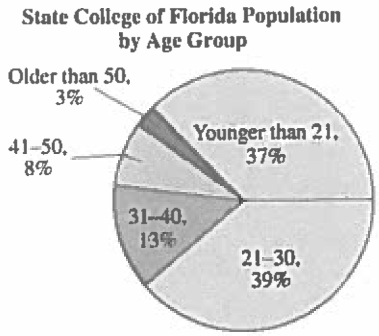
The original compound statement can be written $(p \wedge q) \leftrightarrow r$. By checking the Internet or other references we can find that Vanderbilt University is in Tennesee, Wake Forest University is in North Carolina, and Syracuse University is in New York. Therefore, p is a true statement, but q and r are false statements. We will substitute T for p, F for q, and F for r and evaluate the compound statement:

$$\begin{array}{c}
(p \wedge q) \leftrightarrow r \\
(\mathrm{T} \wedge \mathrm{F}) \leftrightarrow \mathrm{F} \\
\mathrm{F} \quad \leftrightarrow \mathrm{F} \\
\mathrm{T}
\end{array}$$

Therefore, the original compound statement is true. ■

Example 7 *Using Real Data in Compound Statements*

The graph in Fig. 3.2 on page 122 represents the student population by age group in 2009 for the State College of Florida (SCF). Use this graph to determine the truth value of the following compound statements.



Source: www.scf.edu

Figure 3.2

a) If 37% of the SCF population is younger than 21 or 26% of the SCF population is age 21–30, then 13% of the SCF population is age 31–40.

b) 3% of the SCF population is older than 50 and 8% of the SCF population is age 41–50, if and only if 19% of the SCF population is age 21–30.

Solution

a) Let

p: 37% of the SCF population is younger than 21.
q: 26% of the SCF population is age 21–30.
r: 13% of the SCF population is age 31–40.

Then the original statement can be written $(p \vee q) \rightarrow r$. We can see from Fig. 3.2 that both p and r are true statements and that q is a false statement. Substitute T for p, F for q, and T for r and evaluate the statement:

$$(p \vee q) \rightarrow r$$
$$(\text{T} \vee \text{F}) \rightarrow \text{T}$$
$$\text{T} \quad \rightarrow \text{T}$$
$$\text{T}$$

Therefore, "If 37% of the SCF population is younger than 21 or 26% of the SCF population is age 21–30, then 13% of the SCF population is age 31–40" is a true statement.

b) Let

p: 3% of the SCF population is older than 50.
q: 8% of the SCF population is age 41–50.
r: 19% of the SCF population is age 21–30.

Then the original statement can be written $(p \wedge q) \leftrightarrow r$. We can see from Fig. 3.2 that p and q are both true statements and r is a false statement. Substitute T for p, T for q, and F for r and evaluate the statement:

$$(p \wedge q) \leftrightarrow r$$
$$(\text{T} \wedge \text{T}) \leftrightarrow \text{F}$$
$$\text{T} \quad \leftrightarrow \text{F}$$
$$\text{F}$$

Therefore, the original statement, "3% of the SCF population is older than 50 and 8% of the SCF population is age 41–50, if and only if 19% of the SCF population is age 21–30," is a false statement. ■

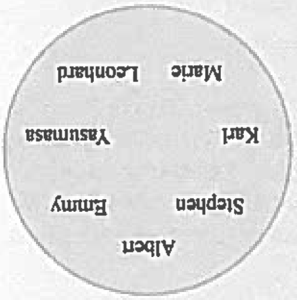
RECREATIONAL MATH

Satisfiability Problems

Suppose you are hosting a dinner party for seven people: Yasumasa, Marie, Albert, Stephen, Leonhard, Karl, and Emmy. You need to develop a seating plan around your circular dining room table that would satisfy all your guests. Albert and Emmy are great friends and must sit together. Yasumasa and Karl haven't spoken to each other in years and cannot sit by each other. Leonhard must sit by Marie or by Albert, but he cannot sit by Karl. Stephen insists he sit by Albert. Can you come up with a plan that would satisfy all your guests? Now imagine the difficulty of such a problem as the list of guests, and their demands, grows.

Problems such as this are known as *satisfiability* problems. The symbolic logic you are studying in this chapter allows computer scientists to represent these problems with symbols and solve the problems using computers. Even with the fastest computers, some satisfiability problems take an enormous amount of time to solve.

One solution to the problem posed above is shown upside down below. Exercises 79 and 80 on page 127 have other satisfiability problems.



Self-Contradictions, Tautologies, and Implications

Two special situations can occur in the truth table of a compound statement: The statement may always be false, or the statement may always be true. We give such statements special names.

Definition: Self-contradiction

A **self-contradiction** is a compound statement that is always false.

When every truth value in the answer column of the truth table is false, then the statement is a self-contradiction.

Table 3.22

p	q	$(p \leftrightarrow q)$	$\wedge$	$(p$	$\leftrightarrow$	$\sim q)$
T	T	T	F	T	F	F
T	F	F	F	T	T	T
F	T	F	F	F	T	F
F	F	T	F	F	F	T
		1	5	2	4	3

Example 8 *All Falses, a Self-Contradiction*

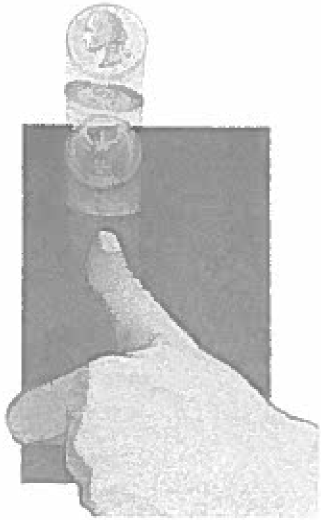
Construct a truth table for the statement $(p \leftrightarrow q) \wedge (p \leftrightarrow \sim q)$.

Solution See Table 3.22. In this example, the truth values are false in each case of column 5. This statement is an example of a self-contradiction or a *logically false statement.* ■

Definition: Tautology

A **tautology** is a compound statement that is always true.

When every truth value in the answer column of the truth table is true, the statement is a tautology.



▲ *"Heads I win, tails you lose." Do you think that this statement is a tautology, self-contradiction, or neither? See Exercise 77 on page 126.*

Example 9 *All Trues, a Tautology*

Construct a truth table for the statement $(p \wedge q) \rightarrow (p \vee r)$.

Solution The answer is given in column 3 of Table 3.23. The truth values are true in every case. Thus, the statement is an example of a tautology or a *logically true statement.*

Table 3.23

p	q	r	$(p \wedge q)$	$\rightarrow$	$(p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	F
			1	3	2

The conditional statement $(p \wedge q) \rightarrow (p \vee r)$ is a tautology. Conditional statements that are tautologies are called *implications.* In Example 9, we can say that $p \wedge q$ *implies* $p \vee r$.

Definition: Implication

An **implication** is a conditional statement that is a tautology.

In any implication the antecedent of the conditional statement implies the consequent. In other words, if the antecedent is true, then the consequent must also be true. That is, the consequent will be true whenever the antecedent is true.

Example 10 *An Implication?*

Determine whether the conditional statement $[(p \wedge q) \wedge p] \rightarrow q$ is an implication.

Solution If the conditional statement is a tautology, the conditional statement is an implication. Because the conditional statement is a tautology (see Table 3.24), the conditional statement is an implication. The antecedent $[(p \wedge q) \wedge p]$ implies the consequent q. Note that the antecedent is true only in case 1 and that the consequent is also true in case 1. ■

Table 3.24

p	q	$[(p \wedge q)$	$\wedge$	$p]$	$\rightarrow$	q
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	F	F	F	T	T
F	F	F	F	F	T	F
		1	3	2	5	4

SECTION 3.3 Exercises

Warm Up Exercises

In Exercises 1–6, fill in the blanks with an appropriate word, phrase, or symbol(s).

1. The conditional statement $p \rightarrow q$ is ________ only when p is true and q is false.

2. In the conditional statement $p \rightarrow q$,

   a) The lower-case letter p represents the ________.

   b) The lower-case letter q represents the ________.

3. The biconditional statement $p \leftrightarrow q$ is ________ only when p and q have the same truth value.

4. A compound statement that is always true is known as a ________.

5. A compound statement that is always false is known as a ________.

6. A conditional statement that is a tautology is known as an ________.

Practice the Skills

In Exercises 7–16, construct a truth table for the statement.

7. $\sim p \rightarrow q$

8. $\sim p \rightarrow \sim q$

9. $\sim(p \rightarrow \sim q)$

10. $\sim(\sim p \leftrightarrow q)$

11. $\sim q \leftrightarrow p$

12. $(p \leftrightarrow q) \rightarrow p$

13. $p \leftrightarrow (q \vee p)$

14. $(\sim q \wedge p) \rightarrow \sim q$

15. $q \rightarrow (p \rightarrow \sim q)$

16. $(p \vee q) \leftrightarrow (p \wedge q)$

In Exercises 17–24, construct a truth table for the statement.

17. $\sim p \rightarrow (q \wedge r)$

18. $q \vee (p \rightarrow \sim r)$

19. $(q \vee \sim r) \leftrightarrow \sim p$

20. $(p \wedge r) \rightarrow (q \vee r)$

21. $(\sim r \vee \sim q) \rightarrow p$

22. $[r \wedge (q \vee \sim p)] \leftrightarrow \sim p$

23. $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim r)$

24. $(\sim p \leftrightarrow \sim q) \rightarrow (\sim q \leftrightarrow r)$

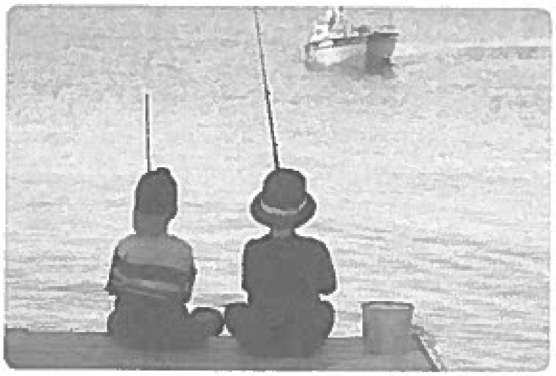
In Exercises 25–30, write the statement in symbolic form. Then construct a truth table for the symbolic statement.

25. If it is raining, then the baseball game is canceled and we can eat dinner together.

26. We advance in the tournament if and only if Max plays, or Pondo does not show up.

27. The election was fair if and only if the polling station stayed open until 8 P.M., or we will request a recount.

28. If the dam holds then we can go fishing, if and only if the pole is not broken.



29. If Mary Andrews does not send me an e-mail then we can call her, or we can write to Mom.

30. It is false that if Eileen Jones went to lunch, then she cannot take a message and we will have to go home.

In Exercises 31–36, determine whether the statement is a tautology, self-contradiction, or neither.

31. $\sim p \rightarrow p$

32. $(\sim p \vee q) \leftrightarrow \sim q$

33. $\sim p \wedge (q \leftrightarrow \sim q)$

34. $(p \wedge \sim q) \rightarrow q$

35. $(\sim q \rightarrow p) \vee \sim q$

36. $[(p \rightarrow q) \vee r] \leftrightarrow [(p \wedge q) \rightarrow r]$

In Exercises 37–42, determine whether the statement is an implication.

37. $\sim p \rightarrow p$

38. $p \rightarrow (p \vee q)$

39. $\sim p \rightarrow \sim (p \wedge q)$

40. $(p \vee q) \rightarrow (p \vee \sim r)$

41. $[(p \rightarrow q) \wedge (q \rightarrow p)] \rightarrow (p \leftrightarrow q)$

42. $[(p \vee q) \wedge r] \rightarrow (p \vee q)$

In Exercises 43–52, if p is true, q is false, and r is true, find the truth value of the statement.

43. $p \rightarrow (q \rightarrow r)$

44. $(p \wedge \sim q) \rightarrow \sim r$

45. $(p \wedge q) \leftrightarrow (q \vee r)$

46. $r \rightarrow (\sim p \leftrightarrow \sim q)$

47. $(\sim p \wedge \sim q) \vee \sim r$

48. $\sim [p \rightarrow (q \wedge r)]$

49. $(\sim p \leftrightarrow r) \vee (\sim q \leftrightarrow r)$

50. $(r \rightarrow \sim p) \wedge (q \rightarrow \sim r)$

51. $\sim [(p \vee q) \leftrightarrow (p \rightarrow \sim r)]$

52. $[(\sim r \rightarrow \sim q) \vee (p \wedge \sim r)] \rightarrow q$

Problem Solving

In Exercises 53–60, determine the truth value for each simple statement. Then, using the truth values, determine the truth value of the compound statement.

53. If $\sqrt{25} = 5$, then $\sqrt{49} = 7$.

54. If $5 < 1$, then $7 > 10$.

55. If a cat has whiskers or a fish can swim, then a dog lays eggs.

56. If Dallas is in Texas and St. Louis is in Missouri, then Detroit is in California.

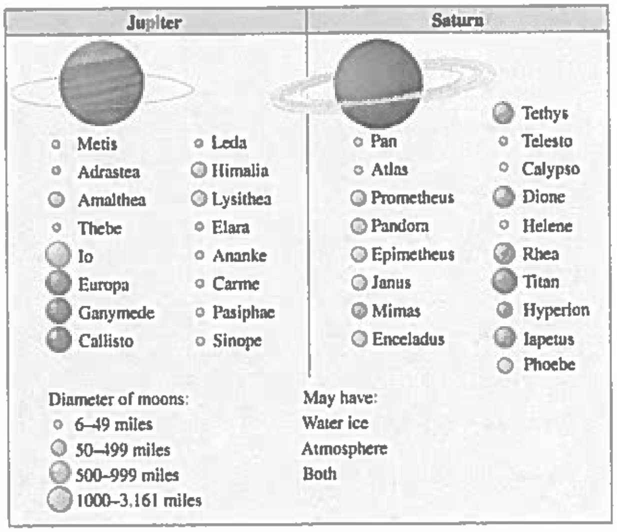
57. Apple makes computers, if and only if Nike makes sports shoes or Rolex makes watches.

58. Spike Lee is a movie director, or if Halle Berry is a school teacher then George Clooney is a circus clown.

59. Mother's Day is in May and Father's Day is in December, if and only if Thanksgiving is in April.

60. Honda makes automobiles or Honda makes motorcycles, if and only if Toyota makes cereal.

In Exercises 61–64, use the information provided about the moons for the planets Jupiter and Saturn to determine the truth values of the simple statements. Then determine the truth value of the compound statement.



Source: Data from *Time* Magazine

61. ***Jupiter's Moons*** Io has a diameter of 1000–3161 miles or Thebe may have water, and Io may have atmosphere.

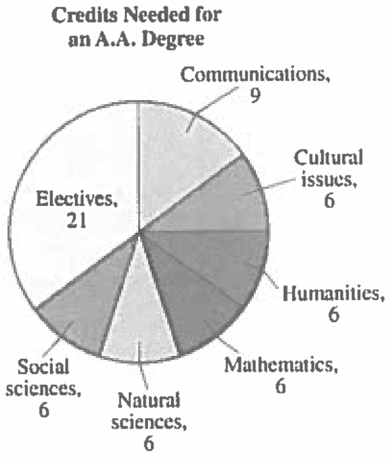
62. ***Moons of Saturn*** Titan may have water and Titan may have atmosphere, if and only if Janus may have water.

63. ***Moon Comparisons*** Phoebe has a larger diameter than Rhea if and only if Callisto may have water ice, and Calypso has a diameter of 6–49 miles.

64. ***Moon Comparisons*** If Jupiter has 16 moons or Saturn does not have 18 moons, then Saturn has 7 moons that may have water ice.

College Credits *In Exercises 65 and 66, use the graph to determine the truth value of each simple statement. Then determine the truth value of the compound statement.*

The following graph shows the number of credits in various categories needed by Jose Silva to earn his Associate in Arts degree from Keiser University.



65. The number of communications credits needed is more than the number of mathematics credits needed and the number of cultural issues credits needed is equal to the number of humanities credits needed, if and only if the number of social sciences credits needed is more than the number of natural sciences credits needed.

66. If the number of elective credits needed is 21 or the number of communications credits needed is 15, then the number of humanities credits needed is equal to the number of mathematics credits needed.

In Exercises 67–72, suppose both of the following statements are false.

p: Muhundan spoke at the teachers' conference.
q: Muhundan received the outstanding teacher award.

Find the truth values of each compound statement.

67. If Muhundan received the outstanding teacher award, then Muhundan spoke at the teachers' conference.

68. If Muhundan spoke at the teachers' conference, then Muhundan did not receive the outstanding teacher award.

69. If Muhundan did not receive the outstanding teacher award, then Muhundan spoke at the teachers' conference.

70. Muhundan did not receive the outstanding teacher award if and only if Muhundan spoke at the teachers' conference.

71. Muhundan received the outstanding teacher award if and only if Muhundan spoke at the teachers' conference.

72. If Muhundan did not receive the outstanding teacher award, then Muhundan did not speak at the teachers' conference.

73. ***A New Computer*** Your parents make the following statement to your sister, "If you get straight A's this semester, then we will buy you a new computer." At the end of the semester your parents buy your sister a new computer. Can you conclude that your sister got straight A's? Explain.

74. ***Job Interview*** Consider the statement "If your interview goes well, then you will be offered the job." If you are interviewed and then offered the job, can you conclude that your interview went well? Explain.

Challenge Problems/Group Activities

In Exercises 75 and 76, construct truth tables for the symbolic statement.

75. $[p \vee (q \rightarrow \sim r)] \leftrightarrow (p \wedge \sim q)$

76. $[(r \rightarrow \sim q) \rightarrow \sim p] \vee (q \leftrightarrow \sim r)$

77. Is the statement "Heads I win, tails you lose" a tautology, a self-contradiction, or neither? Explain your answer.

78. Construct a truth table for

a) $(p \vee q) \rightarrow (r \wedge s)$.

b) $(q \rightarrow \sim p) \vee (r \leftrightarrow s)$.

Recreational Mathematics

79. ***Satisfiability Problem*** Refer to the Recreational Mathematics box on page 122 and then solve the following satisfiability problem. Allen, Booker, Chris, and Dennis all were born in the same year—one in January, one in February, one in March, and one in April. Chris was born before Dennis. Dennis was born two months after Booker. Booker was born after Allen, but before Chris. Find out who was born in each month.

80. ***Cat Puzzle*** Solve the following puzzle. The Joneses have four cats. The parents are Tiger and Boots, and the kittens are Sam and Sue. Each cat insists on eating out of its own bowl. To complicate matters, each cat will eat only its own brand of cat food. The colors of the bowls are red, yellow, green, and blue. The different types of cat food are Whiskas, Friskies, Nine Lives, and Meow Mix. Tiger will eat Meow Mix if and only if it is in a yellow bowl. If Boots is to eat her food, then it must be in a yellow bowl. Mrs. Jones knows that the label on the can containing Sam's food is the same color as his bowl. Boots eats Whiskas. Meow Mix and Nine Lives are packaged in a brown paper bag. The color of Sue's bowl is green if and only if she eats Meow Mix. The label on the Friskies can is red. Match each cat with its food and the bowl of the correct color.

81. ***The Youngest Triplet*** The Barr triplets have an annoying habit: Whenever a question is asked of the three of them, two tell the truth and the third lies. When I asked them which of them was born last, they replied as follows.

Mary: Katie was born last.
Katie: I am the youngest.
Annie: Mary is the youngest.

Which of the Barr triplets was born last?

Internet/Research Activity

82. Select an advertisement from the Internet, a newspaper, or a magazine that makes or implies a conditional statement. Analyze the advertisement to determine whether the consequent necessarily follows from the antecedent. Explain your answer. (See Example 3.)

SECTION 3.4 Equivalent Statements

▲ *Which statement is equivalent to If it is sunny, then we go to the beach?*

Suppose your friend makes the following statements:

If it is sunny, then we go to the beach.

If we go to the beach, then it is sunny.

If it is not sunny, then we do not go to the beach.

If we do not go to the beach, then it is not sunny.

Are these statements all saying the same thing, or does each one say something completely different from the others? In this section, we will study how we can answer this question by using logic symbols and truth tables. We also will learn to identify variations of conditional statements.

Why ***This Is Important*** Understanding when two statements are equivalent is important to understanding advertisers' claims, political statements, and legal documents.

Equivalent statements are an important concept in the study of logic.

Definition: Equivalent

Two statements are **equivalent**, symbolized ⇔,* if both statements have exactly the same truth values in the answer columns of the truth tables.

Sometimes the words ***logically equivalent*** are used in place of the word *equivalent*.

---

*The symbol ≡ is also used to indicate equivalent statements.

To determine whether two statements are equivalent, construct a truth table for each statement and compare the answer columns of the truth tables. If the answer columns are identical, the statements are equivalent. If the answer columns are not identical, the statements are not equivalent.

Example 1 *Equivalent Statements*

Determine whether the following two statements are equivalent.

$$p \wedge (q \vee r)$$
$$(p \wedge q) \vee (p \wedge r)$$

Solution Construct a truth table for each statement (see Table 3.25).

Table 3.25

p	q	r	p	$\wedge$	$(q \vee r)$	$(p \wedge q)$	$\vee$	$(p \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F
T	F	T	T	T	T	F	T	T
T	F	F	T	F	F	F	F	F
F	T	T	F	F	T	F	F	F
F	T	F	F	F	T	F	F	F
F	F	T	F	F	T	F	F	F
F	F	F	F	F	F	F	F	F
			1	3	2	1	3	2

Because the truth tables have the same answer (column 3 for both tables), the statements are equivalent. Therefore, we can write

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

■

Example 2 *Are the Following Equivalent Statements?*

Determine whether the following statements are equivalent.

a) If your homework is finished and you have washed the dishes, then you can watch television.

b) If your homework is not finished or you have not washed the dishes, then you cannot watch television.

Solution First write each statement in symbolic form, then construct a truth table for each statement. If the answer columns of both truth tables are identical, then the statements are equivalent. If the answer columns are not identical, then the statements are not equivalent.

Let

p: Your homework is finished.
q: You have washed the dishes.
r: You can watch television.

In symbolic form, the statements are

a) $(p \wedge q) \rightarrow r$. b) $(\sim p \vee \sim q) \rightarrow \sim r$.

The truth tables for these statements are given in Tables 3.26 and 3.27, respectively. The answers in the columns labeled 5 are not identical, so the statements are not equivalent. ■

Table 3.26

p	q	r	$(p$	$\wedge$	$q)$	$\rightarrow$	r
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	F	F	T	T
T	F	F	T	F	F	T	F
F	T	T	F	F	T	T	T
F	T	F	F	F	T	T	F
F	F	T	F	F	F	T	T
F	F	F	F	F	F	T	F
			1	3	2	5	4

Table 3.27

p	q	r	$(\sim p$	$\vee$	$\sim q)$	$\rightarrow$	$\sim r$
T	T	T	F	F	F	T	F
T	T	F	F	F	F	T	T
T	F	T	F	T	T	F	F
T	F	F	F	T	T	T	T
F	T	T	T	T	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T
			1	3	2	5	4

Example 3 *Which Statements Are Logically Equivalent?*

Determine which statement is logically equivalent to "It is not true that the tire is both out of balance and flat."

a) If the tire is not flat, then the tire is not out of balance.

b) The tire is not out of balance or the tire is not flat.

c) The tire is not flat and the tire is not out of balance.

d) If the tire is not out of balance, then the tire is not flat.

Solution To determine whether any of the choices are equivalent to the given statement, first write the given statement and the choices in symbolic form. Then construct truth tables and compare the answer columns of the truth tables.

Let

p: The tire is out of balance.
q: The tire is flat.

The given statement may be written "It is not true that the tire is out of balance and the tire is flat." The statement is expressed in symbolic form as $\sim(p \wedge q)$. Using p and q as indicated, choices (a) through (d) may be expressed symbolically as

a) $\sim q \rightarrow \sim p$. b) $\sim p \vee \sim q$. c) $\sim q \wedge \sim p$. d) $\sim p \rightarrow \sim q$.

Now construct a truth table for the given statement (Table 3.28) and for each statement (a) through (d), given in Table 3.29 (a) through (d). By examining the truth tables, we see that the given statement, $\sim(p \wedge q)$, is logically equivalent to choice (b), $\sim p \vee \sim q$. Therefore, the correct answer is "The tire is not out of balance or the tire is not flat." This statement is logically equivalent to the statement "It is not true that the tire is both out of balance and flat."

Table 3.28

p	q	$\sim$	$(p$	$\wedge$	$q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	T	F	F	T
F	F	T	F	F	F
		4	1	3	2

Table 3.29

		(a)			(b)			(c)			(d)		
p	q	$\sim q$	$\rightarrow$	$\sim p$	$\sim p$	$\vee$	$\sim q$	$\sim q$	$\wedge$	$\sim p$	$\sim p$	$\rightarrow$	$\sim q$
T	T	F	T	F	F	F	F	F	F	F	F	T	F
T	F	T	F	F	F	T	T	T	F	F	F	T	T
F	T	F	T	T	T	T	F	F	F	T	T	F	F
F	F	T	T	T	T	T	T	T	T	T	T	T	T

■

De Morgan's Laws

Example 3 showed that a statement of the form $\sim(p \wedge q)$ is equivalent to a statement of the form $\sim p \vee \sim q$. Thus, we may write $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$. This equivalent statement is one of two special laws called De Morgan's laws. The laws, named after Augustus De Morgan, an English mathematician, were first introduced in Section 2.4, where they applied to sets.

De Morgan's Laws

1. $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$
2. $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

Profile In Mathematics

Charles Dodgson

Lewis Carroll.

One of the more interesting and well-known students of logic was Charles Dodgson (1832–1898), better known to us as Lewis Carroll, the author of *Alice's Adventures in Wonderland* and *Through the Looking-Glass*. Although the books have a child's point of view, many argue that the audience best equipped to enjoy them is an adult one. Dodgson, a mathematician, logician, and photographer (among other things), uses the naïveté of a 7-year-old girl to show what can happen when the rules of logic are taken to absurd extremes.

"You should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least—at least I mean what I say—that's the same thing, you know."

"Not the same thing a bit!" said the Hatter. "You might as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

You can demonstrate that De Morgan's second law is true by constructing and comparing truth tables for $\sim(p \vee q)$ and $\sim p \wedge \sim q$. Do so now.

When using De Morgan's laws, if it becomes necessary to negate an already negated statement, use the fact that $\sim(\sim p)$ is equivalent to p. For example, the negation of the statement "Today is not Monday" is "Today is Monday."

Example 4 *Use De Morgan's Laws*

Select the statement that is logically equivalent to "I do not have investments, but I do not have debts."

a) I do not have investments or I do not have debts.

b) It is false that I have investments and I have debts.

c) It is false that I have investments or I have debts.

d) I have investments or I have debts.

Solution To determine which statement is equivalent, write each statement in symbolic form.

Let

p: I have investments.

q: I have debts.

The statement "I do not have investments, but I do not have debts" written symbolically is $\sim p \wedge \sim q$. Recall that the word *but* means the same thing as *and*. Now, write parts (a) through (d) symbolically.

a) $\sim p \vee \sim q$ b) $\sim(p \wedge q)$ c) $\sim(p \vee q)$ d) $p \vee q$

De Morgan's law shows that $\sim p \wedge \sim q$ is equivalent to $\sim(p \vee q)$. Therefore, the answer is (c): "It is false that I have investments or I have debts." ■

Example 5 *Using De Morgan's Laws to Write an Equivalent Statement*

Write a statement that is logically equivalent to "It is not true that tomatoes are poisonous or eating peppers cures the common cold."

Solution Let

p: Tomatoes are poisonous.

q: Eating peppers cures the common cold.

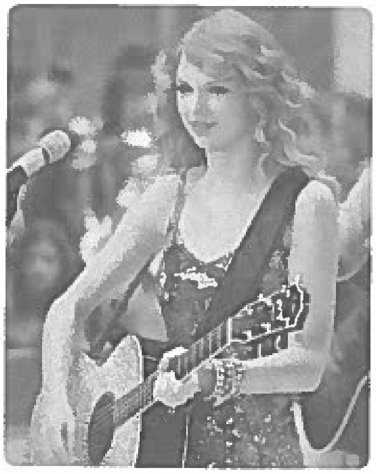
The given statement is of the form $\sim(p \vee q)$. Using the second of De Morgan's laws, we see that an equivalent statement in symbols is $\sim p \wedge \sim q$. Therefore, an equivalent statement in words is "Tomatoes are not poisonous and eating peppers does not cure the common cold." ■

Consider $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$, one of De Morgan's laws. To go from $\sim(p \wedge q)$ to $\sim p \vee \sim q$, we negate both the p and the q within parentheses; change the conjunction, $\wedge$, to a disjunction, $\vee$; and remove the negation symbol preceding the left parentheses and the parentheses themselves. We can use a similar procedure to obtain equivalent statements. For example,

$$\sim(\sim p \wedge q) \Leftrightarrow p \vee \sim q$$
$$\sim(p \wedge \sim q) \Leftrightarrow \sim p \vee q$$

We can use a similar procedure to obtain equivalent statements when a disjunction is within parentheses. Note that

$$\sim(\sim p \vee q) \Leftrightarrow p \wedge \sim q$$
$$\sim(p \vee \sim q) \Leftrightarrow \sim p \wedge q$$



▲ *Taylor Swift*

Example 6 *Using De Morgan's Laws to Write an Equivalent Statement*

Use De Morgan's laws to write a statement logically equivalent to "Taylor Swift did not win an Academy of Country Music award, but she had a top-selling record."

Solution Let

p: Taylor Swift won an Academy of Country Music award.
q: Taylor Swift had a top-selling record.

The statement written symbolically is $\sim p \wedge q$. Earlier we showed that

$$\sim p \wedge q \Leftrightarrow \sim(p \vee \sim q)$$

Therefore, the statement "It is false that Taylor Swift won an Academy of Country Music award or Taylor Swift did not have a top-selling record" is logically equivalent to the given statement. ■

There are strong similarities between the topics of sets and logic. We can see them by examining De Morgan's laws for sets and logic.

De Morgan's laws: set theory	**De Morgan's laws: logic**
$(A \cap B)' = A' \cup B'$	$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$
$(A \cup B)' = A' \cap B'$	$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

The complement in set theory, $'$, is similar to the negation, $\sim$, in logic. The intersection, $\cap$, is similar to the conjunction, $\wedge$; and the union, $\cup$, is similar to the disjunction, $\vee$. If we were to interchange the set symbols with the logic symbols, De Morgan's laws would remain, but in a different form.

Both $'$ and $\sim$ can be interpreted as *not*.

Both $\cap$ and $\wedge$ can be interpreted as *and*.

Both $\cup$ and $\vee$ can be interpreted as *or*.

For example, the set statement $A' \cup B$ can be written as a statement in logic as $\sim a \vee b$.

Profile In Mathematics

Augustus De Morgan

Augustus De Morgan (1806–1871), the son of a member of the East India Company, was born in India and educated at Trinity College, Cambridge (UK). One of the great reformers of logic in the nineteenth century, De Morgan made his greatest contribution to the subject by realizing that logic as it had come down from Aristotle was narrow in scope and could be applied to a wider range of arguments. His work laid the foundation for modern, symbolic logic.

Statements containing connectives other than *and* and *or* may have equivalent statements. To illustrate this point, construct truth tables for $p \rightarrow q$ and for $\sim p \vee q$. The truth tables will have the same answer columns and therefore the statements are equivalent. We summarize this as follows.

The Conditional Statement Written As a Disjunction

$$p \rightarrow q \Leftrightarrow \sim p \vee q$$

With these equivalent statements, we can write a conditional statement as a disjunction or a disjunction as a conditional statement. For example, the statement "If the game is polo, then you ride a horse" can be equivalently stated as "The game is not polo or you ride a horse."

To change a conditional statement to a disjunction, negate the antecedent, change the conditional symbol to a disjunction symbol, and keep the consequent the same. To change a disjunction statement to a conditional statement, negate the first statement, change the disjunction symbol to a conditional symbol, and keep the second statement the same.

Example 7 *Rewriting a Disjunction as a Conditional Statement*

Write a conditional statement that is logically equivalent to "The Oregon Ducks will win or the Oregon State Beavers will lose." Assume that the negation of winning is losing.

Solution Let

p: The Oregon Ducks will win.
q: The Oregon State Beavers will win.

The original statement may be written symbolically as $p \vee \sim q$. To write an equivalent conditional statement, negate the first statement, p, change the disjunction symbol to a conditional symbol, and keep the second statement the same. Symbolically, the equivalent statement is $\sim p \rightarrow \sim q$. The equivalent statement in words is "If the Oregon Ducks lose, then the Oregon State Beavers will lose." ■

Negation of the Conditional Statement

Now we will discuss how to negate a conditional statement. To negate a conditional statement we use the fact that $p \rightarrow q \Leftrightarrow \sim p \vee q$ and De Morgan's laws. Examples 8 and 9 show the process.

Example 8 *The Negation of a Conditional Statement*

Determine a statement equivalent to $\sim(p \rightarrow q)$.

Solution Begin with $p \rightarrow q \Leftrightarrow \sim p \vee q$, negate both statements, and use De Morgan's laws.

$$\begin{aligned} p \rightarrow q &\Leftrightarrow \sim p \vee q && \\ \sim(p \rightarrow q) &\Leftrightarrow \sim(\sim p \vee q) && \text{Negate both statements.} \\ &\Leftrightarrow p \wedge \sim q && \text{De Morgan's laws} \end{aligned}$$

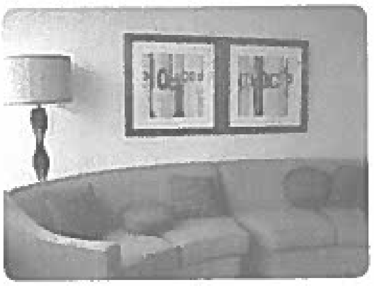
Therefore, $\sim(p \rightarrow q)$ is equivalent to $p \wedge \sim q$. ■

We summarize the result of Example 8 as follows.

The Negation of the Conditional Statement Written As a Conjunction

$$\sim(p \rightarrow q) \Leftrightarrow p \wedge \sim q$$

Example 9 *Write an Equivalent Statement*



Write a statement that is equivalent to "It is false that if you hang the picture then it will be crooked."

Solution Let

p: You hang the picture.
q: The picture will be crooked.

The given statement can be represented symbolically as $\sim(p \rightarrow q)$. We illustrated in Example 8 that $\sim(p \rightarrow q)$ is equivalent to $p \wedge \sim q$. Therefore, an equivalent statement is "You hang the picture and the picture will not be crooked." ■

Using the fact that $\sim(p \rightarrow q) \Leftrightarrow p \wedge \sim q$, can you determine what $\sim(p \rightarrow \sim q)$ is equivalent to as a conjunction? If you answered $p \wedge q$ you answered correctly.

Variations of the Conditional Statement

We know that $p \rightarrow q$ is equivalent to $\sim p \vee q$. Are any other statements equivalent to $p \rightarrow q$? Yes, there are many. Now let's look at the variations of the conditional statement to determine whether any are equivalent to the conditional statement. *The variations of the conditional statement are made by switching and/or negating the antecedent and the consequent of a conditional statement.* The variations of the conditional statement are the ***converse*** of the conditional, the ***inverse*** of the conditional, and the ***contrapositive*** of the conditional.

Listed here are the variations of the conditional with their symbolic form and the words we say to read each one.

Variations of the Conditional Statement

Name	Symbolic form	Read
Conditional	$p \rightarrow q$	"If p, then q"
Converse of the conditional	$q \rightarrow p$	"If q, then p"
Inverse of the conditional	$\sim p \rightarrow \sim q$	"If not p, then not q"
Contrapositive of the conditional	$\sim q \rightarrow \sim p$	"If not q, then not p"

TIMELY TIP
The contrapositive statement is always equivalent to the original conditional statement.

To write the converse of the conditional statement, switch the order of the antecedent and the consequent. To write the inverse, negate both the antecedent and the consequent. To write the contrapositive, switch the order of the antecedent and the consequent and then negate both of them.

Are any of the variations of the conditional statement equivalent? To determine the answer, we can construct a truth table for each variation, as shown in Table 3.30. It reveals that *the conditional statement is equivalent to the contrapositive statement and that the converse statement is equivalent to the inverse statement.*

Table 3.30

p	q	*Conditional* $p \rightarrow q$	*Contrapositive* $\sim q \rightarrow \sim p$	*Converse* $q \rightarrow p$	*Inverse* $\sim p \rightarrow \sim q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

■

Example 10 *The Converse, Inverse, and Contrapositive*

For the conditional statement "If the song contains sitar music, then the song was written by George Harrison," write the

a) converse. b) inverse. c) contrapositive.


▲ *A sitar.*

Solution

a) Let

p: The song contains sitar music.
q: The song was written by George Harrison.

The conditional statement is of the form $p \rightarrow q$, so the converse must be of the form $q \rightarrow p$. Therefore, the converse is "If the song was written by George Harrison, then the song contains sitar music."

b) The inverse is of the form $\sim p \rightarrow \sim q$. Therefore, the inverse is "If the song does not contain sitar music, then the song was not written by George Harrison."

c) The contrapositive is of the form $\sim q \rightarrow \sim p$. Therefore, the contrapositive is "If the song was not written by George Harrison, then the song does not contain sitar music." ■

Example 11 *Determine the Truth Values*

Let

p: The number is divisible by 10.
q: The number is divisible by 5.

Write the following statements and determine which are true.

a) The conditional statement, $p \rightarrow q$
b) The converse of $p \rightarrow q$
c) The inverse of $p \rightarrow q$
d) The contrapositive of $p \rightarrow q$

Solution

a) The conditional statement in symbols is $p \rightarrow q$. Therefore, in words the conditional statement is *If the number is divisible by 10, then the number is divisible by 5.* This statement is true. A number divisible by 10 must also be divisible by 5, since 5 is a divisor of 10.

b) The converse of the conditional statement in symbols is $q \rightarrow p$. Therefore, in words the converse is *If the number is divisible by 5, then the number is divisible by 10.* This statement is false. For example, 15 is divisible by 5, but 15 is not divisible by 10.

c) The inverse of the conditional statement in symbols is $\sim p \rightarrow \sim q$. Therefore, in words the inverse is *If the number is not divisible by 10, then the number is not divisible by 5.* This statement is false. For example, 25 is not divisible by 10, but 25 is divisible by 5.

d) The contrapositive of the conditional statement in symbols is $\sim q \rightarrow \sim p$. Therefore, in words the contrapositive is *If the number is not divisible by 5, then the number is not divisible by 10.* This statement is true. Any number that is not divisible by 5 cannot be divisible by 10, since 5 is a divisor of 10. ■

Because the contrapositive statement is always equivalent to the original conditional statement, in Example 11 d) we should have expected the answer to be a true statement because the original conditional statement was also a true statement.

Example 12 *Use the Contrapositive*

Use the contrapositive to write a statement logically equivalent to "If you don't eat your meat, then you can't have any pudding."

Solution Let

p: You do eat your meat.
q: You can have any pudding.

The given statement written symbolically is

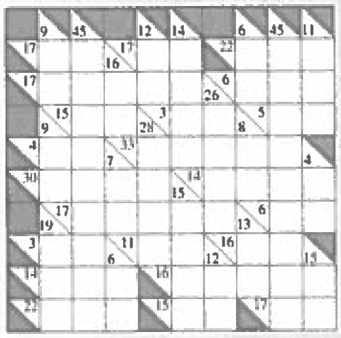
$$\sim p \rightarrow \sim q$$

RECREATIONAL MATH

Kakuro

Like Sudoku (see the *Recreational Mathematics* box on Sudoku on page 97), Kakuro is a puzzle that requires logic. The puzzle resembles a crossword puzzle but uses numbers for clues instead of words. The aim of the game is to fill all the blank squares in the grid with only the digits 1–9 so that the numbers you enter add up to the corresponding clues. For example, in the puzzle shown below, the 17 near the upper left corner of the puzzle is a clue that indicates that the two numbers that you place in the two squares to the right of the 17 must add up to equal 17. The 9 near the upper left corner of the puzzle is a clue that indicates that the two numbers that you place in the two squares below the 9 must add up to equal 9.

In addition, you may not repeat any digits in any given "word" or string of numbers.



For more information and more puzzles, see www.kakuro.com. The solution to the above puzzle can be found in the answer section in the back of this book. For an additional puzzle, see Exercise 83 on page 140.

The contrapositive of the statement is

$$q \rightarrow p$$

Therefore, an equivalent statement is "If you can have any pudding, then you do eat your meat." ■

The contrapositive of the conditional is very important in mathematics. Consider the statement "If a^2 is not a whole number, then a is not a whole number." Is this statement true? You may find this question difficult to answer. Writing the statement's contrapositive may enable you to answer the question. The contrapositive is "If a is a whole number, then a^2 is a whole number." Since the contrapositive is a true statement, the original statement must also be true.

Example 13 *Which Are Equivalent?*

Determine which, if any, of the following statements are equivalent. You may use De Morgan's laws, the fact that $p \rightarrow q \Leftrightarrow \sim p \vee q$, information from the variations of the conditional, or truth tables.

a) If you leave by 9 A.M., then you will get to your destination on time.

b) You do not leave by 9 A.M. or you will get to your destination on time.

c) It is false that you will get to your destination on time or you did not leave by 9 A.M.

d) If you do not get to your destination on time, then you did not leave by 9 A.M.

Solution Let

p: You leave by 9 A.M.
q: You will get to your destination on time.

In symbolic form, the four statements are

a) $p \rightarrow q$. b) $\sim p \vee q$. c) $\sim(q \vee \sim p)$. d) $\sim q \rightarrow \sim p$.

Which of these statements are equivalent? Earlier in this section, you learned that $p \rightarrow q$ is equivalent to $\sim p \vee q$. Therefore, statements (a) and (b) are equivalent. Statement (d) is the contrapositive of statement (a). Therefore, statement (d) is also equivalent to statement (a) and statement (b). Statements (a), (b), and (d) all have the same truth table (Table 3.31).

Table 3.31

		(a)	(b)	(d)
p	q	$p \rightarrow q$	$\sim p \vee q$	$\sim q \rightarrow \sim p$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Now let's look at statement (c). To determine whether $\sim(q \vee \sim p)$ is equivalent to the other statements, we will construct its truth table (Table 3.32) and compare the answer column with the answer columns in Table 3.31.

Table 3.32

				(c)	
p	q	$\sim$	$(q$	$\vee$	$\sim p)$
T	T	F	T	T	F
T	F	T	F	F	F
F	T	F	T	T	T
F	F	F	F	T	T
		4	1	3	2

MATHEMATICS TODAY

Fuzzy Logic

Many modern computers work solely with two values, 1 or 0. This constraint makes it difficult for a computer to evaluate vague concepts that human beings deal with on a regular basis, such as *bright*, *slow*, and *light*. More and more computers are becoming more capable of handling such vague concepts, thanks to the introduction of ***fuzzy logic***. Unlike the traditional computer logic, fuzzy logic is based on the assignment of a value between 0 and 1, inclusively, that can vary from setting to setting. For example, a camera using fuzzy logic may assign *bright* a value of 0.9 on a sunny day, 0.4 on a cloudy day, and 0.1 at night. Fuzzy logic also makes use of logical statements like those studied in this chapter. One such statement is "If X and Y, then Z." For example, a computer chip in a camera programmed with fuzzy logic may use the rule "If the day is *bright* and the film speed is *slow*, then let in less *light*." Fuzzy logic is discussed further in Exercises 81 and 82 on page 139.

Why This Is Important Fuzzy logic is increasingly used in many devices that are part of everyday life.

None of the three answer columns of the truth table in Table 3.31 is the same as the answer column of the truth table in Table 3.32. Therefore $\sim(q \vee \sim p)$ is not equivalent to any of the other statements. Therefore, only statements (a), (b), and (d) are equivalent to each other. ■

SECTION 3.4 Exercises

Warm Up Exercises

In Exercises 1–8, fill in the blanks with an appropriate word, phrase, or symbol(s).

1. Statements that have exactly the same truth values in the answer columns of their truth tables are called ________ statements.

2. DeMorgan's laws state that

   a) $\sim(p \wedge q)$ is equivalent to ________, and

   b) $\sim(p \vee q)$ is equivalent to ________.

3. The conditional statement $p \rightarrow q$ is equivalent to the following disjunction statement: ________.

4. The negation of the conditional statement $\sim(p \rightarrow q)$ is equivalent to the following conjunction statement: ________.

5. Given the conditional statement $p \rightarrow q$, the converse of the conditional statement in symbolic form is ________.

6. Given the conditional statement $p \rightarrow q$, the inverse of the conditional statement in symbolic form is ________.

7. Given the conditional statement $p \rightarrow q$, the contrapositive of the conditional statement in symbolic form is ________.

8. Of the converse, inverse, and contrapositive, only the contrapositive of the conditional statement is ________ to the conditional statement.

Practice the Skills

In Exercises 9–18, use De Morgan's laws to determine whether the two statements are equivalent.

9. $\sim(p \wedge q), \sim p \wedge \sim q$

10. $\sim(p \wedge q), \sim p \vee \sim q$

11. $\sim(p \vee q), \sim p \wedge \sim q$

12. $\sim(p \vee q), \sim p \vee \sim q$

13. $\sim(\sim p \wedge q), p \wedge \sim q$

14. $\sim(\sim p \wedge q), p \vee \sim q$

15. $(\sim p \vee \sim q) \rightarrow r, \sim(p \wedge q) \rightarrow r$

16. $q \rightarrow \sim(p \wedge \sim r), q \rightarrow \sim p \vee r$

17. $\sim(p \rightarrow \sim q), p \wedge q$

18. $\sim(\sim p \rightarrow q), \sim p \wedge \sim q$

In Exercises 19–30, use a truth table to determine whether the two statements are equivalent.

19. $p \rightarrow q, \sim p \vee q$

20. $\sim(p \rightarrow q), p \wedge \sim q$

21. $\sim q \rightarrow \sim p, p \rightarrow q$

22. $q \rightarrow p, \sim p \rightarrow \sim q$

23. $(p \vee q) \vee r, p \vee (q \vee r)$

24. $p \vee (q \wedge r), \sim p \rightarrow (q \wedge r)$

25. $p \wedge (q \vee r), (p \wedge q) \vee r$

26. $\sim(q \rightarrow p) \vee r, (p \vee q) \wedge \sim r$

27. $(p \rightarrow q) \wedge (q \rightarrow p), (p \leftrightarrow q)$

28. $[\sim(p \rightarrow q)] \wedge [\sim(q \rightarrow p)], \sim(p \leftrightarrow q)$

Problem Solving

In Exercises 29–34, use De Morgan's laws to write an equivalent statement for the sentence.

29. It is false that Oregon borders the Atlantic Ocean and Delaware borders the Pacific Ocean.

30. It is false that Greg Dietrich selects formulas or Moana Karsteter applies concepts.

31. The dog was neither a bulldog nor was the dog a boxer.

32. The pot roast is hot, but it is not well done.

33. If Ashley Tabai takes the new job, then she will not move or she will buy a new house in town.

34. If Phil Murphy buys us dinner, then we will not go to the top of the CN Tower but we will be able to walk to the Red Bistro Restaurant.

In Exercises 35–40, use the fact that $p \rightarrow q$ is equivalent to $\sim p \vee q$ to write an equivalent form of the given statement.

35. If Janette Campbell buys a new car, then she sells her old car.

36. Byron Dyce did not walk to the meeting or we started late.

37. Bob the Tomato visited the nursing home or he did not visit the Cub Scout meeting.

38. If Joanne Ernst goes to the Lightning game, then she will not go to the Rays game.

39. Chase is not hiding or the pitcher is broken.

40. If Weezer is not on the radio, then Tim Ollendick is working.

In Exercises 41–48, use the fact that $\sim(p \rightarrow q)$ is equivalent to $p \wedge \sim q$ to write the statement in an equivalent form.

41. It is false that if we go to Chicago, then we will go to Navy Pier.

▲ *Navy Pier in Chicago*

42. It is false that if General Electric makes the telephone, then the telephone is made in the United States.

43. I am cold and the heater is not working.

44. The Badgers beat the Nittany Lions and the Bucks beat the 76ers.

45. It is not true that if Amazon has a sale then we will buy $100 worth of books.

46. Thompson is sick today but Allen didn't go to school.

47. John Deere will hire new workers and the city of Dubuque will retrain the workers.

48. My cell phone is not a Blackberry but my cell phone has a keyboard.

In Exercises 49–54, write the converse, inverse, and contrapositive of the statement. (For Exercise 54, use De Morgan's laws.)

49. If Nanette Berry teaches macramé, then she needs extra yarn.

50. If the water is running, then Linus is getting a drink.

51. If I go to Mexico, then I buy silver jewelry.

52. If Bob Dylan records a new CD, then he will go on tour.

53. If that annoying paper clip shows up on my computer screen, then I will scream.

54. If the sun is shining, then we will go down to the marina and we will take out the sailboat.

In Exercises 55–60, write the contrapositive of the statement. Use the contrapositive to determine whether the conditional statement is true or false.

55. If a natural number is divisible by 14, then the natural number is divisible by 7.

56. If the quadrilateral is a parallelogram, then the opposite sides are parallel.

57. If a natural number is divisible by 3, then the natural number is divisible by 6.

58. If $1/n$ is not a natural number, then n is not a natural number.

59. If two lines do not intersect in at least one point, then the two lines are parallel.

60. If $\frac{m \cdot a}{m \cdot b} \neq \frac{a}{b}$, then m is not a counting number.

In Exercises 61–74, determine which, if any, of the three statements are equivalent (see Example 13).

61. a) If the ball lands in foul territory, then the runner returns to the base.
    b) If the runner returns to the base, then the ball lands in foul territory.
    c) The runner does not return to the base or the ball lands in foul territory.

62. a) If Fido is our dog's name, then Rex is not our dog's name.

b) It is false that Fido is our dog's name and Rex is not our dog's name.

c) Fido is not our dog's name or Rex is our dog's name.

63. a) The office is not cool and the copier is jammed.

b) If the office is not cool, then the copier is not jammed.

c) It is false that the office is cool or the copier is not jammed.

64. a) The test is not written or the review sheet is not ready.

b) If the test is written, then the review sheet is ready.

c) It is false that the review sheet is ready and the test is not written.

65. a) Today is not Sunday or the library is open.

b) If today is Sunday, then the library is not open.

c) If the library is open, then today is not Sunday.

66. a) If you are fishing at 1 P.M., then you are driving a car at 1 P.M.

b) You are not fishing at 1 P.M. or you are driving a car at 1 P.M.

c) It is false that you are fishing at 1 P.M. and you are not driving a car at 1 P.M.

67. a) The grass grows and the trees are blooming.

b) If the trees are blooming, then the grass does not grow.

c) The trees are not blooming or the grass does not grow.

68. a) Johnny Patrick is chosen as department chair if and only if he is the only candidate.

b) If Johnny Patrick is chosen as department chair then he is the only candidate, and if Johnny Patrick is the only candidate then he is chosen as department chair.

c) Johnny Patrick is not chosen as department chair and he is not the only candidate.

69. a) If the corn bag goes in the hole, then you are awarded three points.

b) It is false that the corn bag goes in the hole and you are awarded three points.

c) The corn bag does not go in the hole and you are not awarded three points.

70. a) Fitz and the Tantrums will not go on tour if and only if James King does not play the saxophone.

b) It is false that Fitz and the Tantrums will go on tour if and only if James King does not play the saxophone.

c) If Fitz and the Tantrums go on tour, then James King plays saxophone.

71. a) If the pay is good and today is Monday, then I will take the job.

b) If I do not take the job, then it is false that the pay is good or today is Monday.

c) The pay is good and today is Monday, or I will take the job.

72. a) If you are 18 years old and a citizen of the United States, then you can vote in the presidential election.

b) You can vote in the presidential election, if and only if you are a citizen of the United States and you are 18 years old.

c) You cannot vote in the presidential election, or you are 18 years old and you are not a citizen of the United States.

73. a) The package was sent by Federal Express, or the package was not sent by United Parcel Service but the package arrived on time.

b) The package arrived on time, if and only if it was sent by Federal Express or it was not sent by United Parcel Service.

c) If the package was not sent by Federal Express, then the package was not sent by United Parcel Service but the package arrived on time.

74. a) The mortgage rate went down, if and only if Tim purchased the house and the down payment was 10%.

b) The down payment was 10%, and if Tim purchased the house then the mortgage rate went down.

c) If Tim purchased the house, then the mortgage rate went down and the down payment was not 10%.

Concept/Writing Exercises

75. If p and q represent two simple statements, and if $p \rightarrow q$ is a false statement, what must be the truth value of the converse, $q \rightarrow p$? Explain.

76. If p and q represent two simple statements, and if $p \rightarrow q$ is a false statement, what must be the truth value of the inverse, $\sim p \rightarrow \sim q$? Explain.

77. If p and q represent two simple statements, and if $p \rightarrow q$ is a false statement, what must be the truth value of the contrapositive, $\sim q \rightarrow \sim p$? Explain.

78. If p and q represent two simple statements, and if $p \rightarrow q$ is a true statement, what must be the truth value of the contrapositive, $\sim q \rightarrow \sim p$? Explain.

Challenge Problems/Group Activities

79. We learned that $p \rightarrow q \Leftrightarrow \sim p \vee q$. Determine a conjunction that is equivalent to $p \rightarrow q$. (*Hint:* There are many answers.)

80. Determine whether $\sim[\sim(p \vee \sim q)] \Leftrightarrow p \vee \sim q$. Explain the method(s) you used to determine your answer.

81. In an appliance or device that uses fuzzy logic, a change in one condition causes a change in a second condition. For example, in a camera, if the brightness increases, the lens aperture automatically decreases to get the proper exposure. Name at least 10 appliances or devices that make use of fuzzy logic and explain how fuzzy logic is used in each appliance or device. See the *Mathematics Today* box on page 135.

82. In symbolic logic, a statement is either true or false (consider true to have a value of 1 and false a value of 0). In fuzzy logic, nothing is true or false, but everything is a matter of degree. For example, consider the statement "The sun is shining." In fuzzy logic, this statement may have a value between 0 and 1 and may be constantly changing. For example, if the sun is partially blocked by clouds, the value of this statement may be 0.25. In fuzzy logic, the values of connective statements are found as follows for statements p and q.

Not p has a truth value of $1 - p$.

$p \wedge q$ has a truth value equal to the lesser of p and q.

$p \vee q$ has a truth value equal to the greater of p and q.

$p \rightarrow q$ has a truth value equal to the lesser of 1 and $1 - p + q$.

$p \leftrightarrow q$ has a truth value equal to $1 - |p - q|$, that is, 1 minus the absolute value* of p minus q.

Suppose the statement "p: The sun is shining" has a truth value of 0.25 and the statement "q: Mary is getting a tan" has a truth value of 0.20. Find the truth value of

a) $\sim p$

b) $\sim q$

c) $p \wedge q$

d) $p \vee q$

e) $p \rightarrow q$

f) $p \leftrightarrow q$

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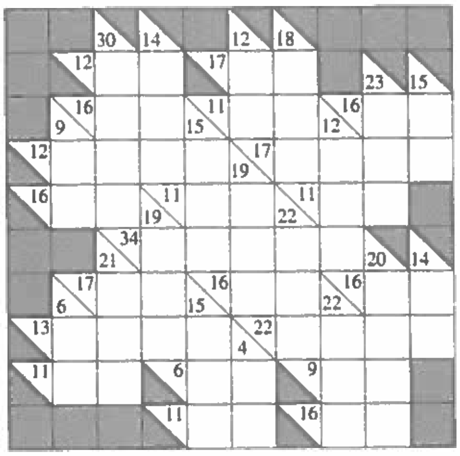
*Absolute values are discussed in Section 13.8.

Recreational Exercises

83. ***Kakuro*** Refer to the *Recreational Mathematics* box on page 134. Complete the following Kakuro puzzle.



Internet/Research Activities

84. Do research and write a report on fuzzy logic.

85. Read one of Lewis Carroll's books and write a report on how he used logic in the book. Give at least five specific examples.

86. Do research and write a report on the life and achievements of Augustus De Morgan. Indicate in your report his contributions to sets and logic.

SECTION 3.5 Symbolic Arguments

▲ *The Who plays during halftime of Super Bowl XLIV.*

Consider the following statements.

If Carrie Underwood sings the national anthem, then The Who will play at halftime.

Carrie Underwood sings the national anthem.

If you accept these two statements as true, then what logical conclusion could you draw? Do you agree that you can logically conclude that The Who will play at halftime? In this section, we will use our knowledge of logic to study the structure of such statements to draw logical conclusions.

Why ***This is Important*** Drawing logical conclusions from a given set of statements is an important application of logic. Also, we use logic daily in decision making after considering all the facts we know to be true.

Previously in this chapter, we used symbolic logic to determine the truth value of a compound statement. We now extend those basic ideas to determine whether we can draw logical conclusions from a set of given statements. Consider once again the two statements.

If Carrie Underwood sings the national anthem, then The Who will play at halftime.

Carrie Underwood sings the national anthem.

These statements in the following form constitute what we will call a *symbolic argument*.

Premise 1:	If Carrie Underwood sings the national anthem, then The Who will play at halftime.
Premise 2:	Carrie Underwood sings the national anthem
Conclusion:	The Who will play at halftime.