MAC 2233, Spring 2014: Test 2 Review
Exam covers Lectures 11 – 18, Sections 3.1 – 4.2

This review includes typical exam problems. It is not designed to be comprehensive, but to be representative of the topics covered on the exam. You may also pick up the Fall 2013 exam at Broward Teaching center. Remember you have 90 minutes for your exam, so you should pay attention to time as you practice for the exam.

1. Evaluate \( \lim_{x \to -4} f(x) \) if \( f(x) = \frac{x + 4}{\sqrt{x^2 + 3x - 2}} \).

2. Find the value of \( k \) so that \( f(x) = \begin{cases} x^2 - \ln x + 2k & x \leq e \\ x^2 - x & x > e \end{cases} \)

   is continuous for all positive numbers.

3. If \( f(x) = \begin{cases} \frac{x^2 - 16}{x^2 + 3x - 4} & x \neq -4 \\ 0 & x = -4 \end{cases} \)

   find \( p = \lim_{x \to -4^+} f(x) \) and \( q = \lim_{x \to 1^-} f(x) \).

4. The Intermediate Value Theorem guarantees a solution to the equation \( x^3 - \frac{1}{x} + 3 = 5x \) on which of the following intervals?

   a) \([-1, 1]\)  
   b) \([1, 3]\)  
   c) \([3, 5]\)  
   d) \([-3, -2]\)

5. If \( f(x) = \frac{x^3 + 3x^2 + 2x}{x - x^3} \), find a) \( \lim_{x \to 0^+} f(x) \) b) \( \lim_{x \to -1^+} f(x) \) c) \( \lim_{x \to 1^-} f(x) \)

   and d) \( \lim_{x \to -\infty} f(x) \). List all discontinuities and describe as infinite, jump, or removable. Find each vertical and horizontal asymptote of \( f(x) \).

6. If \( f(x) = \frac{|1 - x|}{x^2 - x} \), find a) \( \lim_{x \to 0^-} f(x) \) and b) \( \lim_{x \to 1^+} f(x) \). Find and describe each discontinuity of \( f(x) \) (jump, infinite or removable). Sketch the graph of \( f(x) \).
7. If \( f(x) = \frac{2}{e^{-x} - 3} \), find:

1) \( \lim_{x \to -\infty} f(x) \)  
2) \( \lim_{x \to +\infty} f(x) \)  
3) Each asymptote of the graph of \( f(x) \).

8. Consider the function \( f(x) = \begin{cases} 
  e^x + 1 & x < 0 \\
  2 - x & 0 < x < 2 \\
  \ln(x - 2) & x > 2 
\end{cases} \)

(a) Sketch the graph of \( f(x) \).
(b) Find the following limits if possible.
\[
\lim_{x \to -\infty} f(x) = \quad \lim_{x \to 0^+} f(x) =
\]
\[
\lim_{x \to 0^-} f(x) = \quad \lim_{x \to 1} f(x) =
\]
\[
\lim_{x \to 2^+} f(x) = \quad \lim_{x \to 2^-} f(x) =
\]
(c) List all discontinuities of \( f(x) \) and state whether they are removable or nonremovable.

9. Consider a function \( f(x) \) which has the following graph.

(a) On which interval(s) is \( f(x) \) continuous?
(b) \( f(x) \) has its jump discontinuity/discontinuities at \( x = \)
(c) \( f(x) \) has its infinite discontinuity/discontinuities at \( x = \).
(d) \( f(x) \) has its removable discontinuity/discontinuities at \( x = \).
(e) How would you define or redefine \( f(x) \) at the point(s) in part (d) in order to make \( f(x) \) continuous at the point(s)?
(f) Find each value at which \( f(x) \) is continuous but not differentiable.
(g) Find \( f'(-1) \).  
(h) Which is larger, \( f'(-5) \) or \( f'(-3) \)?
10. Find the value of \( a \) so that the tangent line to \( y = x^2 - 2\sqrt{x} + 1 \) is perpendicular to the line \( ay + 2x = 2 \) when \( x = 4 \).

11. Use the definition of derivative at a point \( f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \) to evaluate: \( \lim_{h \to 0} \frac{(32 + h)^{4/5} - 16}{h} \). Hint: find \( f \) and \( a \), and use a derivative rule to evaluate.

12. (a) Use the definition of derivative to find \( f'(x) \) if \( f(x) = \frac{x}{2x - 1} \). Check your answer using the Quotient Rule.

(b) Find each interval over which \( f(x) \) is differentiable.

(c) Write the equation of the tangent line to \( f(x) = \frac{x}{2x - 1} \) at \( x = -1 \).

13. Indicate whether each of the following statements is true or false.

(a) If \( f \) is continuous at \( x = a \), then \( f \) is differentiable at \( x = a \).

(b) If \( f \) is not continuous at \( x = a \), then \( f \) is not differentiable at \( x = a \).

(c) If \( f \) has a vertical tangent line at \( x = a \), then \( \frac{df}{dx} = 0 \) at \( x = a \).

14. Let \( f(x) = \begin{cases} 2 - x|x| & x < 0 \\ 3x + 2 & x \geq 0 \end{cases} \).

(a) Use the limit definition of continuity to show that \( f(x) \) is continuous at \( x = 0 \).

(b) Find \( f'(0) \) if possible using the limit definition of derivative at a point.

15. Find each value at which \( f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2 \) has a horizontal tangent line.

16. If \( f(x) = (x^3 - 2x)(2\sqrt{x} + 1) \), find \( f'(x) \) two ways: rewriting \( f(x) \) and differentiating, and using the Product Rule.

17. Suppose that for functions \( f \) and \( g \), \( f(2) = -3 \), \( g(2) = 2 \), \( f'(2) = -1 \) and \( g'(2) = 1 \).

(a) Find \( F'(2) \) if \( F(x) = x^2 f(x) \).

(b) Find \( H'(2) \) if \( H(x) = \frac{g(x)}{f(x) + 1} \).
18. Find \( f'(x) = \frac{(\sqrt{x} - 1)^2}{x} \) and write as a single fraction. Write the equation of the tangent line to \( f(x) \) at \( x = 4 \).

19. The demand function for a certain product is given by \( p(x) = -0.02x + 400 \), \( 0 \leq x \leq 20,000 \), where \( p \) is the unit price when \( x \) items are sold.

   (a) Find the revenue function \( R(x) \). Find the marginal revenue when \( x = 300 \). What happens when \( x = 15,000 \)?

   (b) Suppose the cost function for the product is \( C(x) = 100x + 300,000 \). Find the profit function. What is the marginal profit when \( x = 2000 \)?

   (c) Find the actual profit from the sale of the 2001st item. Compare to your answer in (b).

   (d) Find the average profit and marginal average profit when \( x = 100 \).

20. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height \( h \) in feet above the ground after \( t \) seconds is given by \( h(t) = 200 + 64t - 16t^2 \). Find the following:

   (a) The average velocity of the object from time \( t = 0 \) until it reaches its maximum height (hint: consider the graph of the function)

   (b) The instantaneous velocity of the object at time \( t = 1 \) second using the limit definition

21. Sketch a possible graph of the derivative of the function shown below.