# Undergraduate Mathematics Research Symposium University of Florida, Department of Mathematics ${\rm April}\ 28,\,2023$

# Schedule

# Session 1

9:00-9:15	Nicholas Van Nimwegen	Unique Longest Increasing Subsequences in 132-Avoiding Permutations
9:20-9:35	Jianda Du, Nicholas Kapsos, and Joshua Mott	Regression and Feature Analysis for Predicting Wordle Game Scores
9:40-9:55	Rohan Rao	Brief Overview of Lagrange Interpolation
10:00-10:15	Xinyi Zhang	Mathematical Assessment of the Impact Insecticide-Treated Nets (ITNs) on Malaria Dynamics
10:20 -10:40	Coffee break	

# Session 2

10:40-10:55	Haben Belai, Ben Sherwin, and Chris Prieto	On the Stress levels of Astronauts
11:00-11:15	Sai Sivakumar	Toric varieties given by principal 2-minor ideals
11:20-11:35	John McDonald, Trey Black, and Jacob Boesger	Modeling Wordle Difficulty and Player Behavior Using Combined Normalized Difficulty Metric and Neural Net- works
11:40-11:55	Alan Curtin	What is a group of automorphisms?
12:00-12:05	Konstantina Christodoulopoulou and Sara Pollock	Closing remarks
12:05 -1:00		Lunch

## Abstracts

#### Session 1

#### 1.1 Nicholas Van Nimwegen

Mentor: Miklós Bóna (UF, Mathematics)

Title: Unique Longest Increasing Subsequences in 132-Avoiding Permutations

**Abstract:** Given a permutation  $p = p_1 p_2 ... p_n \in \mathfrak{G}_n$ , a set of indices  $i_1 < i_2 < ... < i_k$  is said to define an increasing subsequence if  $p_{i_1} < p_{i_2} < ... < p_{i_k}$ . A permutation is said to have a Unique Longest Increasing Subsequence (ULIS) if there is exactly 1 increasing subsequence of maximal length k. We discuss some of the theory behind ULISs in several classes of permutations as an introduction, and then proceed to the main result. We provide a proof of an open question, that the proportion of 132 avoiding permutations of length n with a ULIS is greater than or equal to 0.5 for all n.

#### 1.2 Jianda Du, Nicholas Kapsos, and Joshua Mott

Mentor: Tracy Stepien (UF, Mathematics)

**Title:** Regression and Feature Analysis for Predicting Wordle Game Scores

**Abstract:** In this talk, we present a model for predicting the likelihood of correctly solving a round of the popular puzzle game Wordle. We discuss our method of feature extraction from historical Wordle results to estimate the perceived difficulty of new words as well as our framework for fitting score distributions to our constructed metrics. We conclude with examples of several test words across the difficulty spectrum, including an interactive tool to compare a user's scores with the predicted outcomes. This talk should be accessible to those without any formal knowledge of statistics.

#### 1.3 Rohan Rao

Mentor: Sara Pollock (UF, Mathematics)

**Title:** Brief Overview of Lagrange Interpolation

**Abstract:** In Numerical Analysis, Lagrange interpolations are use to approximate function using curves, or they can be used to generate an interpolating polynomial that passes through a given set of points. When they are used to approximate a function, Lagrange interpolants have an error formula and the placement of the points at which they are evaluated can play a significant role in how much error is incurred. In order to minimize the error, chebyshev points can be used, which spread out the error mire evenly and minimize the error.

# 1.4 Xinyi Zhang

Mentor: Calistus Ngonghala

**Title:** Mathematical Assessment of the Impact Insecticide-Treated Nets (ITNs) on Malaria Dynamics

**Abstract:** We develop and use a novel dynamic model to assess the impact of ITNs and human

behavior on malaria prevalence and control. The model differs from other published models in that it accounts for 1) human choice to use ITNs properly (for protection) or improperly (for other purposes) through a game theory approach and 2) the decay in ITN efficacy due to natural and human-induced wear. Additionally, the model is structured in terms of individuals who own and use ITNs properly and individuals who do not own or own but misuse ITNs. The model will be extended to other vector-borne diseases, e.g., the Zika virus. Well-formulated and parametrized models can provide valuable insights into disease dynamics and are helpful for public health decision-making.

## Session 2

#### 2.1 Haben Belai, Ben Sherwin, and Chris Prieto

Mentor: David Burrell and Tracy Stepien (UF, Mathematics)

Title: On the Stress levels of Astronauts

**Abstract:** As part of the annual SIMIODE Challenge Using Differential Equations Modeling (SCUDEM) competition, our team tackled the 'Introducing Stress' problem. We used differential equations to model the stress levels of the astronauts in the International Space Station (ISS) as a function of workload, rest time, and duration of stay. Based on our analysis, we gave recommendations to maximize their productivity and minimize stress.

#### 2.2 Sai Sivakumar

Mentor: Ashley Wheeler (Georgia Tech, Mathematics)

**Title:** Toric varieties given by principal 2-minor ideals

**Abstract:** We study affine and projective varieties given by the vanishing of the ideal generated by the principal 2-minors of an  $n \times n$  matrix  $X = (x_{ij})$  of variables. As affine varieties, we prove they are normal but not smooth, and as projective varieties, we prove they are normal, separated, compact, not smooth for n > 2, and not orbifolds for n > 2. We obtain these results by exploiting the toric structure of these varieties through analyzing the convex polyhedral objects associated with them.

#### 2.3 John McDonald, Trey Black, and Jacob Boesger

Mentor: Tracy Stepien (UF, Mathematics)

**Title:** Modeling Wordle Difficulty and Player Behavior Using Combined Normalized Difficulty Metric and Neural Networks

**Abstract:** In this study, we introduce the Combined Normalized Difficulty Metric (CNDM) to classify Wordle solutions as "Easy," "Medium," or "Difficult," based on six quantified attributes. We developed five neural networks to predict the difficulty metrics, the CNDM and its associated error, and to classify words based on difficulty. Additionally, we designed an algorithm to model player strategy and predict guess distribution, and created a statistical model to estimate player population and the proportion of players in Hard Mode. Our analysis revealed an inverse relationship between word difficulty and total player submissions, a direct relationship between word difficulty and Hard Mode submissions, and the impact of major sporting events on player engage-

ment.

#### 2.4 Alan Curtin

Mentor: Dana Bartošová (UF, Mathematics)

**Title:** What is a group of automorphisms?

Abstract: In general, an automorphism is a bijective structure-preserving map whose inverse is also a structure-preserving map. Familiar examples of structure-preserving maps include group homomorphisms, linear transformations on vector spaces and monotone maps on linear orders. What these three structures have in common is that they all are sets equipped with relations or functions. This insight gives rise to the concepts of the first-order structure and automorphisms on first-order structures. As expected, the set of automorphisms on a first-order structure forms a group under function composition. Furthermore, putting the topology of pointwise convergence on the group of automorphisms makes the group operations continuous, allowing us to study groups of automorphisms as topological groups.

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