Undergraduate Mathematics Research Symposium

University of Florida, Department of Mathematics

April 26, 2024

Schedule

Morning Sessions

Session 1

9:00-9:15	Erik Shute	Surface Theory and Soliton Equations
9:20-9:35	Connor Panish	Coalescing Ballistic Annihilation
9:40-9:55	Jianda Du Armando Albornoz	Exploring the Hausdorff and Gromov-Hausdorff Distance
10:00-10:15	Iman Kazmi Diego Espinosa Youssef Kamel	Lamprey's True Enemy Might Be Their Own Biology
10:20 -10:35	Coffee break	

Session 2

10:35-10:50	Michael Poole	Analysis of Entities Less Than Any Real Magnitude
10:55-11:10	Audrey Edwards John McDonald Olutimilehin Sobanjo	Irrational Behavior: A Mathematical Approach to Modeling Aggression and Responses to Aggression
11:15-11:30	Martin Wall	The Gromov–Hausdorff Distance Between Quotient Metric Spaces
11:35-11:50	Oscar Camargo David Castellanos Sharan Majumder	Modeling Environmentally Triggered Sex Determination in Sea Lamprey Populations
11:55-12:10	Kevin Jin	Exploring K-factor in Elo Rating Systems

Lunch: 12:15-1:10

Afternoon Sessions

Session 3		
1:10-1:25	Saisudharshan (Sai) Sivakumar	Free and Virtual Resolutions
1:30-1:45	Tuyen Troung Ziwen Zhu	Modeling Kangaroo Mother Care (KMC): Results from a Model Analysis and Health Outcomes
1:50-2:05	Alejandro Leon	Bounds on the Gromov-Hausdorff Distance between Unit Disk and its Boundary Unit Circle
2:10 -2:25	Coffee break	
Session 4		
2:25-2:40	Samuel Kim	Minimal Flows of the Automorphism Group of the Random Graph
2:45-3:00	Natalie Nicole Perez Rong Fang Megan Sin	Using Differential Equations Modeling to Predict How People's Likelihood to Prefer Retributive Punishment over Restorative Punishment Will Change Over Time
3:05-3:20	Ryan Campisi	Burnside's Lemma for Analysis of Error-Correcting Codes

Abstracts

Session 1

1.1 Erik Shute

Mentor: Luca Di Cerbo

Title: Surface Theory and Soliton Equations

Abstract: We will discuss the relationship between surfaces of constant negative curvature and the sine-Gordon equation, a nonlinear partial differential equation with applications in condensed matter physics. We describe how this relationship arises naturally from geometrical considerations, and how classical transformations of surfaces generate families of sine-Gordon solutions, including so-called soliton solutions. Lastly, we will briefly mention how the surface theory discussed is broadly applicable to many soliton equations.

1.2 Connor Panish

Mentor: Matt Junge (Baruch College)

Title: Coalescing Ballistic Annihilation

Abstract: During the 1980's, the statistical physics literature introduced a model called ballistic annihilation that mimics chemical reactions. Particles in this model are placed randomly throughout the real line and then proceed to move at predetermined discrete velocities. Upon collision, the particles involved are annihilated i.e., removed from the system. In 2019, Haslegrave, Sidoravicius, and Tournier characterized initial conditions for which all particles are eventually annihilated. Expanding upon HST, we study a more general version of this system where particles can survive annihilation with fixed probabilities that are dependent on the velocities of the colliding particles. We prove a formula for the initial conditions necessary for all particles to be annihilated based on the probabilities of survival. Our arguments make use of recursion and hidden symmetries within infinite sums.

1.3 Armando Albornoz and Jianda Du

Mentor: Henry Adams

Title: Exploring the Hausdorff and Gromov-Hausdorff Distance

Abstract: Metrics spaces are ubiquitous but critical in all fields of mathematical studies, including TDA, Topology, Numerical Analysis, etc. We will discuss different notions of similarity between metric spaces and exhibit the importance of considering Hausdorff distance. Finally, we will introduce the Gromov-Hausdorff distance and elucidate the link with the Hausdorff distance through examples that we found in our research and show why it's a good notion of similarity.

1.4 Diego T. Espinosa, Youssef M. Kamel and Iman Kazmi

Mentors: Tracy Stepien and Hemaho Taboe

Title: Lamprey's True Enemy Might Be Their Own Biology

Abstract: Sea lampreys are a type of jawless parasitic fish. The following paper and the mathematical model derived through heavy literature review and consideration, observe how the adaptive sex ratio variation in the sea lamprey population can be monitored and controlled based on its advantages and disadvantages. The parasitic behavior of lampreys often poses a challenge to fisheries and ecosystem stability because of their invasive nature. As we narrow our investigatory scope to the location of the Great Lakes, we synthesize data/information from local monitoring agencies as well as scholarly papers. Our model incorporates variables including population sizes and ratios for each sex, availability of resources in the ecosystem, and reproductive success for each sex. By carefully considering certain parameters and purposefully neglecting others, our analysis provides a realistic model for lasting applications.

Session 2

2.1 Michael Poole

Mentor: Michael Jury

Title: Analysis of Entities Less Than Any Real Magnitude

Abstract: Throughout its history, calculus has been heuristically treated via infinitesimals and infinites, quantities smaller and larger than any real number, respectively. However, no infinitesimals or infinites exist in the real numbers, otherwise such a real infinitesimal would be less than half. 17th century mathematicians knew that infinitesimals and infinites were fictitious, and discouraged the use of these quantities in formulating the standard theory of analysis. In this presentation, we will discuss how to construct a non-Archimedean field that contains the real numbers and these desired infinitesimals and infinites. This field is known as the hyperreal numbers, and it is used to recast elementary calculus and analysis in a more intuitive light.

2.2 Audrey Edwards, John McDonald and Olutimilehin Sobanjo

Mentor: Tracy Stepien

Title: Irrational Behavior: A Mathematical Approach to Modeling Aggression and Responses to Aggression

Abstract: This study investigates the dynamics of different subpopulations of humans based on the manner in which they react when faced with aggressive behavior, as well as the stability of these subpopulations over time based on the type of societal environment they are placed in. A multivariate Lotka-Volterra model was used to run simulations with parameters adjusted to represent when retributive versus meditative punishment is the dominant societal norm in response to aggressive behavior.

2.3 Martin Wall

Mentor: Henry Adams

Title: The Gromov–Hausdorff Distance Between Quotient Metric Spaces

Abstract: The Hausdorff distance measures how far apart two sets are in a common metric space. By contrast, the Gromov–Hausdorff distance provides a notion of distance between two arbitrary

metric spaces. Fix a group G, and suppose that two metric spaces X and Y are each equipped with an action of G by isometries. Then for the Hausdorff distance, if X and Y are G-invariant subspaces of a common metric space, we have that $d_H(X,Y) = d_H(X/G, Y/G)$. We will show, however, that the ratio $\frac{d_{GH}(X/G,Y/G)}{d_{GH}(X,Y)}$ between Gromov–Hausdorff distances can be made both arbitrarily large and arbitrarily small, demonstrating that while the Hausdorff distance is inflexible under quotients, the Gromov–Hausdorff distance is inconstant.

2.4 Oscar Camargo, David Castellanos and Sharan Majumder

Mentor: Tracy Stepien

Title: Modeling Environmentally Triggered Sex Determination in Sea Lamprey Populations

Abstract: When modeling the population dynamics of lampreys, it is crucial to consider that their sex ratio fluctuates with the available resources in their environment. We adapted a nonlinear system of differential equations originally used for modeling Green Sea Turtle sex determination. Our results led us to hypothesize about the significance of environmental sex determination in lampreys and its potential impact on the broader ecosystem.

2.5 Kevin Jin

Mentor: Miklós Bóna

Title: Exploring K-factor in Elo Rating Systems

Abstract: The Elo rating system assigns players ratings to measure their relative skill levels. In this talk, we investigate the K-factor, a variable respon- sible for determining the sensitivity of ratings to change. In practice, it is set arbitrarily based off heuristics. We seek to optimize the K-factor by minimizing the expected error between true skill and skill estimated by rating. Using calculus and convolutions, we try to obtain a closed form expression for error as a function of K-factor. Using this, we can study how different K values impact the accuracy of ratings over time. We also show evidence for our theoretical results using Monte Carlo computer simulation.

Session 3

3.1 Sai Sivakumar

Mentors: Christine Berkesch and Michael Perlman, University of Minnesota, Twin Cities

Title: Free and Virtual Resolutions

Abstract: A syzygy, or free resolution, is a particular sequence of homomorphisms of free modules terminating at some module under investigation. They provide useful information about the module, such as the structure of the module, or geometric information if this module is the coordinate ring of some variety. Virtual resolutions provide similar information, but for Cox rings of smooth toric varieties. We will briefly discuss free resolutions obtained for certain stable submodules of polynomial rings under the action of the general linear group, and short virtual resolutions obtained for Cox rings of finite sets of points in a product of two projective spaces. These are joint works with Dr. Michael Perlman, Bjorn Cattell-Ravdal, Erin Delargy, Akash Ganguly, and Sean Guan, then separately with Dr. Christine Berkesch, Isidora Bailly-Hall, Karina Dovgodko, Sean Guan,

and Jishi Sun.

3.2 Ziwen Zhu and Tuyen Truong

Mentor: Tracy Stepien

Title: Modeling Kangaroo Mother Care (KMC): Results from a Model Analysis and Health Outcomes

Abstract: We develop a differential model to analyze the impact of Kangaroo Mother Care (KMC) treatment on infant mortality rates. Following the standardized definition of KMC provided by the World Health Organization (WHO), we establish the factors in our model as: the duration of skin-to-skin contact and birth weight. We improve the existing logistic differential model by accounting for diminishing returns and developing a separate equation for skin-to-skin contact. Using data from the World Data Bank in 2022, we calibrate our model's constants accordingly. Our analysis suggests that increasing skin-to-skin contact duration can reduce the risk of infant mortality under KMC treatment.

3.3 Alejandro Leon

Mentor: Henry Adams

Title: Bounds on the Gromov-Hausdorff Distance between Unit Disk and its Boundary Unit Circle

Abstract: The Gromov-Hausdorff distance is a way of quantifying the dissimilarity between metric spaces. We investigate the Gromov-Hausdorff distance between the unit disk and its boundary, the unit circle, under the Euclidean metric. We derive upper and lower bounds on this quantity.

Session 4

4.1 Samuel Kim

Mentor: Dana Bartošová

Title: Minimal Flows of the Automorphism Group of the Random Graph

Abstract: It is known that the orbit of the universal minimal flow of the automorphism group of the random graph (Aut (G)) is all linear orderings of the random graph, which has a comeagre orbit of the order type of the random ordered graph, and that this has exactly 42 reducts. We aim to find the minimal flows of Aut(G) using the reducts of the random ordered graph using invariant, closed, equivalence relations of linear orders acting on the random graph.

4.2 Natalie Nicole Perez, Rong Fang and Megan Sin

Mentor: Tracy Stepien

Title: Using Differential Equations Modeling to Predict How People's Likelihood to Prefer Retributive Punishment over Restorative Punishment Will Change Over Time

Abstract: Different people from different backgrounds, lifestyles, and experiences have differing propensities for wanting to punish others for crimes or misdeeds. Based on said experiences, some

will choose to deal with criminals using retributive punishment over restorative punishment, and vice versa. A model was created to view how the different populations of people with different propensities to act out against transgressors changes over time. This model focuses on an individual's wealth as a defining factor due to how it influences both their lifestyle and experiences. With this, a better understanding of how these groups of people make up and affect society was achieved.

4.3 Ryan Campisi

Mentor: Libin Rong

Title: Burnside's Lemma for Analysis of Error-Correcting Codes

Abstract: This research presents an investigation into the structural symmetries and equivalences of error-correcting codes, crucial for maintaining data integrity in transmission and storage. We will delve into the use of Burnside's Lemma from group theory to analyze code words' relationships under symmetry operations. By focusing on the formation of code word orbits under group actions, this research highlights the utility of these concepts through various illustrative examples.

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