(a) $R$ is a ring.
(b) $yx = xy + xy'$ and $y = x(x + x')$.
(c) $R$ is simple. (Hin: Let $f$ be a nonzero element in an ideal $I$ of $R$; then either $f$ has no term involving $y$ or $g = x'f$ is a nonzero element of $I$ that has lower degree than does $f$. In the latter case, consider $xg = gx$. Eventually, find a nonzero $a$ of $f$, which is free of $y$. If $f$ is nonconstant, consider $by = yf$. In a finite number of steps, obtain a nonzero constant element of $I$, hence $I = R$.)
(d) $R$ has no zero divisors.
(e) $R$ is not a division ring.
2. (a) If $A$ is an $R$-module, then $A$ is also a well-defined $R/(a)$-module with $(r + a)(a) = ra + a(r + a)$.
(b) If $A$ is a simple left $R$-module, then $A/\langle a \rangle$ is a primitive ring.
3. Let $V$ be an infinite dimensional vector space over a division ring $D$. (a) If $P$ is the set of all $Hom(D,P)$ such that $I$ is finite dimensional, then $P$ is a proper ideal of the ring $V$. Therefore $Hom(D,P)$ is not simple.
(b) $P$ is itself a simple ring.
(c) $P$ is contained in every nonzero ideal of $Hom(D,P)$.
(d) $Hom(D,P)$ is not (id) Amitsur.