Volume of Sphere

Archimedes (287-212 BC) was the first to discover the volume of the sphere. He had a diagram of a sphere inscribed in a right circular cylinder inscribed on his tombstone to commemorate what he considered his greatest discovery. In a letter he described the method of discovering the formula by visualizing a mechanical balance involving a solid circular cylinder, a solid right circular cone, and a solid sphere. Below is a diagram and the derivation.

Archimedes did not consider this a *proof* of the volume of the sphere and resorted to an elaborate proof using the *method of exhaustion* due to Eudoxus which was the standard at the time. He did recommend his mechanical analysis as a valuable technique for discovering formulas which then had to be verified by an accepted method of proof.

The approach uses an identity to show that the shaded disks just balance. The circle $x^2 + y^2 = 2ax$ gives rise to the identity, $\pi x^2 + \pi y^2 = \pi 2ax$. This gives rise to $2a(\pi x^2 + \pi y^2) = x\pi(2a)^2$ showing that the disks just balance assuming that the connecting arms are massless and that the fulcrum is at the left face of the cylinder. Since the masses and center of masses are known for the cylinder and the cone, the only unknown mass is that of the sphere. For the figures to balance sphere must have volume $V = \frac{4}{3}\pi a^3$.