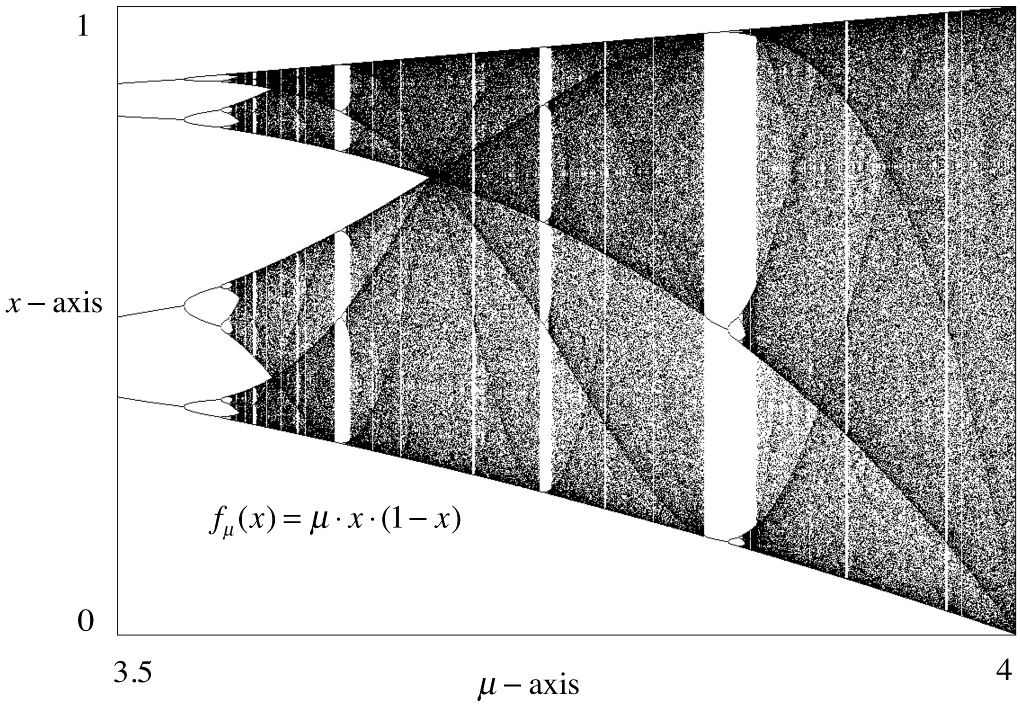
Bifurcation Diagram for the Quadratic Family



The diagram is the bifurcation diagram for the quadratic family, $f_\mu(x) = \mu \cdot x \cdot (1-x)$. The graph is for μ in the range $\frac{7}{2} \le \mu \le 4$ and $0 \le x \le 1$. The diagram represents the long-term behavior of the function $f_\mu(x)$. It is created by taking a particular value of μ and starting with the point $x_0 = \frac{1}{2}$. Iterate the function for 500 times without plotting. Then plot the points $\left\{\left(\mu, f_\mu^n(x_0)\right) \mid n = 501-1000\right\}$. Do the same thing for a large number of equidistributed values of μ in the interval $\frac{7}{2} \le \mu \le 4$, say for 1000 points. The result of the plot is the diagram above.

In class we will show what is happening in the diagram. For instance, why are there *spectral lines* in the diagram? These blank spaces are where there are attracting periodic points. What are the dark regions? These are values of μ for which there are no attracting periodic points.

Some of the most prestigious and talented mathematicians of our time have devoted a major portion of their careers to studying this diagram. It is just one of many major discoveries in mathematics in our time.