In this post we give a proof of the Cauchy Mean Value Theorem. It is a very simple proof and only assumes Rolle’s Theorem.

**Cauchy Mean Value Theorem** Let \( f(x) \) and \( g(x) \) be continuous on \([a, b]\) and differentiable on \((a, b)\). Then there is a \( a < c < b \) such that

\[
(f(b) - f(a)) \cdot g'(c) = (g(b) - g(a)) \cdot g'(c).
\]

**Proof.** The case that \( g(a) = g(b) \) is easy. So, assume that \( g(a) \neq g(b) \). Define

\[
h(x) = f(x) - \frac{f(b) - f(a)}{g(b) - g(a)} \cdot g(x).
\]

Clearly, \( h(a) = h(b) \). Applying Rolle’s Theorem we have that there is a \( c \) with \( a < c < b \) such that

\[
h'(c) = f'(c) - \frac{f(b) - f(a)}{g(b) - g(a)} \cdot g'(c).
\]

For this \( c \) we have that

\[
(f(b) - f(a)) \cdot g'(c) = (g(b) - g(a)) \cdot f'(c).
\]

\[\square\]

The classical Mean Value Theorem is a special case of Cauchy’s Mean Value Theorem. It is the case when \( g(x) \equiv x \). The Cauchy Mean Value Theorem can be used to prove **L’Hospital’s Theorem**.