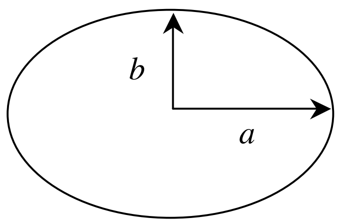
Cavalieri's Determination of the Area of an Ellipse

The principle is the following. Suppose that two planar figures have the same height and at the same level the cross-sectional lengths are in the same ratio *r*. Then the areas are in the same ratio *r*.

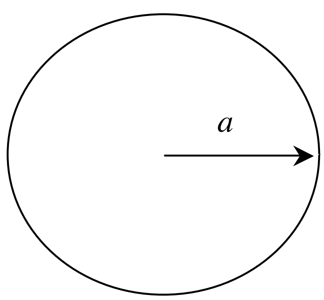
Cavalieri applied this to determine the area of an ellipse. In standard form, the equation of an ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$



Compare this area with that of a circle with radius *a* whose area is πa^2.

$$x^2+y^2=a^2$$



At *x* the cross-sectional lengths are $2y=\frac{b}{a}\sqrt{a^2-a^2}$ and $2y=\sqrt{a^2-x^2}$, respectively. Thus the ratios of these lengths is $\frac{b}{a}$. From this we get that the area of the ellipse is

$$A_{\text{ellipse}}=\frac{b}{a}\cdot\pi a^2=\pi ab.$$