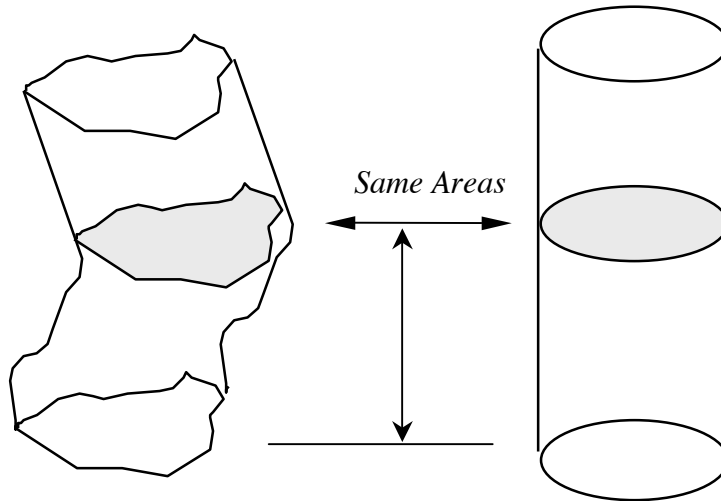
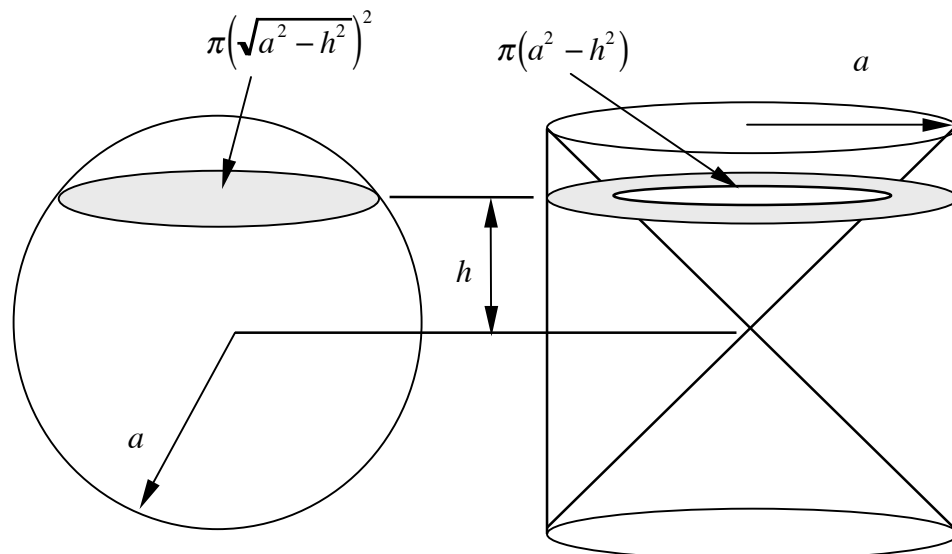


### *Cavalieri's Determination of the Volume of the Sphere*

Bonaventura Cavalieri (1598-1647) was a contemporary of Galileo who considered him the greatest geometer since Archimedes. One of his powerful tools has been called *Cavalieri's Principle* which states that if two solids have the same height and at each level have the same cross sectional area, then they both have the same volume. This is represented with a mnemonic diagram below.



Now we give the diagram that yields a formula for the volume of the sphere. The figure on the left is the sphere. The figure on the right is a cylinder with two cones hollowed out of the top and bottom.



Since the cross sections at each level have the same area, the volumes of the two figures are the same. However, the volume of the figure on the right is easily computed to be  $\pi a^2 \cdot 2a - 2 \cdot \left(\frac{\pi a^3}{3}\right) = \frac{4}{3} \pi a^3$  which is therefore the volume of the sphere.