MAA 4211 FINAL - JAMES KEESLING

NAME ____

Work all problems. Each problem is worth 10 points. Partial credit will be given for correct reasoning even if the final answer is incorrect. Credit will be deducted for incorrect reasoning even if the final answer is correct.

Problem 1. Suppose that X and Y are metric spaces and that $A \subset X$ is compact. Suppose that $f : X \to Y$ is continuous. Show that $f(A)$ is compact.

Problem 2. Suppose $f : [a, b] \to [a, b]$ is continuous and x_0 is a point having period 3. How many points of period 7 are there? How many orbits of period 7? How many points of period 31 are there? How many orbits of period 31. Note that 31 is a prime number.

Problem 3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and suppose that $f(z) = 0$. Suppose also that $f'(z) \neq 0$. Define $g(x) = x - \frac{f(x)}{f'(x)}$. Show that z is an attracting fixed point for $g(x)$.

Problem 4. Show that for every $x \in \mathbb{R}$ there is an integer M such that $M > x$.

Problem 5. Consider $f(x) = x^2 \cdot \sin\left(\frac{1}{x}\right)$ for all $x \neq 0$. Define $f(0) = 0$. Show that $f(x)$ is differentiable for all $x \in \mathbb{R}$ and that $f'(x)$ is not continuous at 0.

Problem 6. State and prove the Banach Fixed Point Theorem.

Problem 7. Suppose that X is a compact metric space and that $f : X \to \mathbb{R}$ is continuous. Show that there is an $x_0 \in X$ such that for all $x \in X$, $f(x_0) \geq f(x)$.

Problem 8. Show that $\sum_{i=0}^{\infty} x^n = \frac{1}{1-x}$ for all $|x| < 1$.

Problem 9. Suppose that X is a complete metric space and that $A \subset X$ is closed. Show that A is a complete metric space.

Problem 10. Show that the Cantor Set C is uncountable.