MAA 4211 PRACTICE TEST 2 - JAMES KEESLING

The problems below are typical of what will be on Test 2. However, the test will consist of only five problems.

Problem 1. Suppose that X and Y are metric spaces and that $A \subset X$ is compact. Suppose that $f : X \to Y$ is continuous. Show that $f(A)$ is compact.
Problem 2. Suppose that X and Y are metric spaces and that $f : X \to Y$ is continuous. Show that if A is connected in X, then $f(A)$ is connected in Y.
Problem 3. Suppose $f : [a, b] \to [a, b]$ is continuous and x_0 is a point having period 3. How many points of period 5 are there? How many orbits of period 5? How many points of period 29 are there?
Problem 4. Let $U \subset \mathbb{R}^n$ be a connected open set. Suppose that $x, y \in U$. Show that there is a continuous $f : [0, 1] \to U$ such that $f(0) = x$ and $f(1) = y$.
Problem 5. Show that $\mathbb{R}$ is complete. Show that $\mathbb{Q}$ is not complete.
Problem 6. State and prove the Banach Fixed Point Theorem.
Problem 7. Suppose that X is a metric space and that $A \subset X$ is compact. Show that A is also complete.
Problem 8. Show that if $f : \mathbb{R} \to \mathbb{R}$ is differentiable at a point x_0, then f is continuous at x_0.
Problem 9. Suppose that X is a compact metric space and that $f : X \to \mathbb{R}$ is continuous. Show that there is an $x_0 \in X$ such that for all $x \in X$, $f(x_0) \geq f(x)$.
Problem 10. Consider $f(x) = x^2 \cdot \sin\left(\frac{1}{x}\right)$ for all $x \neq 0$. Define $f(0) = 0$. Show that $f(x)$ is differentiable for all $x \in \mathbb{R}$ and that $f'(x)$ is not continuous at 0.
Problem 11. Prove the Mean Value Theorem.
Problem 12. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and suppose that $f(z) = 0$. Suppose also that $f'(z) \neq 0$. Define $g(x) = x - \frac{f(x)}{f'(x)}$. Show that z is an attracting fixed point for g.
Problem 13. Determine numerical solutions of the following equations.

$$\cos(x) = x$$

$$x^{15} + 3x^4 - 4x^3 + x = 5$$

Problem 14. Prove the Taylor Remainder Theorem.