FALL 2017 QUIZ 1

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Problem 2. State the **Least Upper Bound Property** for the real numbers.

Problem 3. Show that for every $x \in \mathbb{R}$ there is an integer M such that $M > x$.

Problem 4. Show that for every $\varepsilon > 0$, there is a positive integer n such that $\frac{1}{n} < \varepsilon$.

Problem 5. Let $\{x_i\}_{i=1}^{\infty}$ be a sequence of real numbers. Suppose that $z \in \mathbb{R}$. What does $\lim_{i \to \infty} x_i = z$ mean? We say that $\{x_i\}_{i=1}^{\infty}$ **converges** to z and that the sequence is **convergent**. We say that z is the **limit point** for the sequence.

Problem 6. Suppose that $\{x_i\}_{i=1}^{\infty}$ is a sequence of real numbers such that for all $i < j$, $x_i \leq x_j$. We say that $\{x_i\}_{i=1}^{\infty}$ is **monotone non-decreasing**. Suppose also that the sequence is bounded above. Show that the sequence is convergent. What is the limit?

Problem 7. Show that $\lim_{n \to \infty} \frac{1}{n} = 0$.

Problem 8. Show that $\lim_{n \to \infty} x^n = 0$ for all $0 < x < 1$. Show this for all $|x| < 1$.

Problem 9. Show that $\sum_{i=0}^{\infty} x^n = \frac{1}{1-x}$ for all $|x| < 1$.

Problem 10. Show that $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$. If you were trying to determine if the series converges on your calculator, what conclusion might you arrive at? What limit might the series seem to have?

Problem 11. Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges. Can you determine the limit?

Problem 12. Show that every nonnegative real number x has a decimal expansion, $x = a_0.a_1, a_2, \cdots$ where $a_0 \in \mathbb{N} \cup \{0\}$ and $a_i \in \{0, 1, 2, \ldots, 9\}$ for all $i > 0$. Are the decimal representations unique? When does a non-negative real number have more than one decimal representation? When does a decimal representation represent a rational number?