FALL 2017 QUIZ 2

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Problem 1. State the **Bolzano-Weiststraas Theorem**.

Problem 2. Define what it means for a $A \subset \mathbb{R}$ to be compact. Given an example of a compact set. Give an example of a set which is not compact.

Problem 3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous. Show that if A is compact, then $f(A)$ is also compact.

Problem 4. Suppose that A is compact and that $B \subset A$ is closed. Show that B ia also compact.

Problem 5. Suppose that $\{x_i\}_{i=1}^{\infty}$ is a sequence of real numbers such that for all $i < j$, $x_i \leq x_j$. We say that $\{x_i\}_{i=1}^{\infty}$ is **monotone non-decreasing**. Suppose also that the sequence is bounded above. Show that the sequence is convergent. What is the limit?

Problem 6. Suppose that $A_1 \supset A_2 \supset \cdots A_n \supset A_{n+1} \supset \cdots$ Suppose that each A_i is closed. Show that $\cap_{i=1}^{\infty} A_i$ is closed.

Problem 7. In Problem 6 show that if each A_i is compact then $\cap_{i=1}^{\infty} A_i$ is compact.