

MAA 4211 FALL 2017 QUIZ 3

JAMES KEESLING

Problem 1. Suppose that $f : [a, b] \rightarrow \mathbb{R}$ is continuous. Show that there is a $c \in [a, b]$ such that $f(c) \geq f(x)$ for all $x \in [a, b]$

Problem 2. Let $\{A_i\}_{i=1}^{\infty}$ be a sequence of compact sets in \mathbb{R} . Suppose that This collection has the **Finite Intersection Property**. Show that $\bigcap_{i=1}^{\infty} A_i \neq \emptyset$.

Problem 3. Suppose that A and B are disjoint compact subsets of \mathbb{R} . Show that there is an $a \in A$ and a $b \in B$ such that for all $x \in A$ and all $y \in B$, $d(x, y) \geq d(a, b)$.

Problem 4. Give an example of two disjoint closed sets A and B in \mathbb{R} such that the distance between the sets is $d(A, B) = 0$.

Problem 5. Let $f : A \rightarrow \mathbb{R}$ be a function. We say that f is **Uniformly Continuous** provided that for every $\epsilon > 0$, there is a $\delta > 0$ such that for every x and y in A , if $d(x, y) < \delta$, then $d(f(x), f(y)) < \epsilon$. Show that if A is compact and $f : A \rightarrow \mathbb{R}$ is continuous, then $f : A \rightarrow \mathbb{R}$ is uniformly continuous.