MAA 4211 TEST 1 - JAMES KEESLING

NAME ____

Work all problems. Each problem is worth 20 points. Partial credit will be given for correct reasoning even if the final answer is incorrect. Credit will be deducted for incorrect reasoning even if the final answer is correct.

Problem 1. Use the **Least Upper Bound Property** of the real numbers to show that if $a \in \mathbb{R}$, then there is an integer $n \in \mathbb{N}$ such that $n > a$. This is known as **Archimedes' Principle**.

Problem 2. Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

Problem 3. Show that if $0 < x < 1$, then $\lim_{n\to\infty} x^n = 0$.

Problem 4. Let $C = \{x = \sum_{n=1}^{\infty} \frac{a_n}{3^n} \mid a_n \in \{0,2\} \ \ n \in \mathbb{N}\}$ be the **Cantor Set**. Show that there is a continuous function $f : C \to [0,1]$ which is onto.

Problem 5. Show that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for all $|x| < 1$.