Problem 1. State and prove the Contraction Mapping Theorem.

Problem 2. Show that the following power series converges to the function \( \ln(1 + x) \).

\[
\ln(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}
\]

For what values of \( x \) does the power series converge? Explain.
Problem 3. Consider the differential equation
\[
\frac{dx}{dt} = \sin(t) \cdot x \quad x(0) = \pi.
\]
Use four iterations of **Picard Iteration** to determine a function that estimates the solution.

\[x_0 = \pi\]
\[x_1 =\]
\[x_2 =\]
\[x_3 =\]
\[x_4 =\]

Problem 4. What is the centroid of the graph of the curve \(y = x^2\) over \([0, 1]\)? What is the centroid of the area bounded by this curve over \([0, 1]\)?
Problem 5. Determine the volume of a torus using Pappus’ Theorem. Assume that the torus is formed by rotating a disk of radius \( b \) around the \( y \)-axis with the center of the disk a distance \( a \) from the axis with \( a > b \).

Problem 6. Suppose that \( f : [a, b] \to \mathbb{R} \) is monotone. Show that \( \int_a^b f(x)dx \) exists.
Problem 7. State the Fundamental Theorem of Calculus.

Problem 8. Use Romberg integration with $2^5$ intervals to estimate the following integral. How accurate is the estimate?

$$\int_1^2 \sin(x^2)dx$$
Problem 9. Let $f(x)$ be defined as below.

$$f(x) = \begin{cases} 
1 & x \in \mathbb{Q} \\
0 & x \notin \mathbb{Q} 
\end{cases}$$

Show that $f(x)$ is not Riemann Integrable on $[0, 1]$.

Problem 10. State the Baire Category Theorem. Use the Baire Category Theorem to show that there are real numbers that are not rational numbers.