MAA 4212 – KEESLING – FINAL

Name ____

Work all problems and show all work. Each problem is worth 10 points. Partial credit will be given for correct reasoning even though the final answer may be wrong. Credit will be deducted for incorrect work even though the answer may be right.

Problem 1. State and prove the **Contraction Mapping Theorem**.

Problem 2. Show that the the following power series converges to the function $\ln(1+x)$.

$$\ln(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

For what values of x does the power series converge? Explain.

Problem 3. Consider the differential equation

$$\frac{dx}{dt} = \sin(t) \cdot x \qquad x(0) = \pi.$$

Use four iterations of **Picard Iteration** to determine a function that estimates the solution.

$$x_0 = \pi$$

$$x_1 =$$

$$x_2 =$$

$$x_3 =$$

$$x_4 =$$

Problem 4. What is the centroid of the graph of the curve $y = x^2$ over $[0,1]$? What is the centroid of the area bounded by this curve over $[0,1]$?

Problem 5. Determine the volume of a torus using **Pappus' Theorem** Assume that the torus is formed by rotating a disk of radius b around the $y-$axis with the center of the disk a distance a from the axis with $a > b$.

Problem 6. Suppose that $f : [a, b] \to \mathbb{R}$ is monotone. Show that $\int_a^b f(x)dx$ exists.

Problem 7. State the **Fundamental Theorem of Calculus**.

Problem 8. Use Romberg integration with 2^5 intervals to estimate the following integral. How accurate is the estimate?

$$\int_1^2 \sin(x^2)dx$$

Problem 9. Let $f(x)$ be defined as below.

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$$

Show that $f(x)$ is not **Riemann Integrable** on $[0, 1]$.

Problem 10. State the **Baire Category Theorem**. Use the Baire Category Theorem to show that there are real numbers that are not rational numbers.