MAA 4212 – KEESLING – FINAL

Name ______________________________________

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning even though the final answer may be wrong. Credit will be deducted for incorrect work even though the answer may be right.

Problem 1. State and prove the Contraction Mapping Theorem.

Problem 2. Show that the the following power series converges to the function $\arctan(x)$.

$$\arctan(x) = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n+1}}{2n+1}$$

For what values of x does the power series converge? Explain.

Problem 3. Consider the differential equation

$$\frac{dx}{dt} = \sin(t) \cdot \cos(x) \qquad x(0) = \pi.$$

Find the polynomial of sixth degree, $a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6$, that best approximates the solution of this differential equation. Give the exact coefficients. [Hint, use the TaylorMethod program downloaded onto your calculator.]

Problem 4. What is the centroid of the curve $y = x^2$ over $[0, 1]$? What is the centroid of the area bounded by this curve over $[0, 1]$?

Problem 5. Determine the volume of a torus using Pappus' Theorem Assume that the torus is formed by rotating a disk of radius b around the $y-$axis with the center of the disk a distance a from the axis with $a > b$.

Problem 6. Suppose that $f : [a, b] \to \mathbb{R}$ is monotone. Show that $\int_a^b f(x)dx$ exists.

Problem 7. Suppose that $f(x)$ is continuous on $[a, b]$. Show that $f(x)$ is uniformly continuous on $[a, b]$.

Problem 8. Use Romberg integration with 2^5 intervals to estimate the following integral. How accurate is the estimate?

$$\int_1^2 \sin(x^2)dx$$

Problem 9. Let $f(x)$ be defined as below.

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$$

Show that $f(x)$ is not Riemann integrable on $[0, 1]$.

Problem 10. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable and that $f'(x)$ is bounded on $\mathbb{R}$. Show that $f(x)$ is uniformly continuous on $\mathbb{R}$.