MAA 4212 QUIZ

Problem 1. Suppose that $f(x) = \sum_{n=0}^{\infty} a_n(x-a)^n$. Suppose that the radius of convergence of this series is $R > 0$. Show that $a_n = \frac{f^{(n)}(a)}{n!}$ for all $n \in \mathbb{N}$.

Problem 2. What is the radius of convergence of the series $f(x) = \sum_{n=0}^{\infty} 2^n(x-3)^n$? What is the interval of convergence for this series?

Problem 3. Show that the power series for $\cos(x) = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{(2k)!}$ converges to $\cos(x)$ for all $x \in \mathbb{R}$.

Problem 4. Show that the power series for $\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges to $\exp(x)$ for all $x \in \mathbb{R}$.

Problem 5. Let $f(x) = \sqrt{1+x}$. What is the power series for this function centered at $a = 0$ and for what values of x does it converge? Can you show that the series converges to $\sqrt{1+x}$ on its interval of convergence?