Problem 1. Suppose that \( f(x) = \sum_{n=0}^{\infty} a_n(x-a)^n \). Suppose that the radius of convergence of this series is \( R > 0 \). Show that \( a_n = \frac{f^{(n)}(a)}{n!} \) for all \( n \in \mathbb{N} \).

Problem 2. What is the radius of convergence of the series \( f(x) = \sum_{n=0}^{\infty} 2^n(x-3)^n \)? What is the interval of convergence for this series?

Problem 3. Show that the power series for \( \cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} \) converges to \( \cos(x) \) for all \( x \in \mathbb{R} \).

Problem 4. Show that the power series for \( \exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \) converges to \( \exp(x) \) for all \( x \in \mathbb{R} \).

Problem 5. Let \( f(x) = \sqrt{1+x} \). What is the power series for this function centered at \( a = 0 \) and for what values of \( x \) does it converge? Can you show that the series converges to \( \sqrt{1+x} \) on its interval of convergence?