MAA 4212 TEST 1 – JAMES KEESLING

NAME _____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. Suppose that $f : [a, b] \to \mathbb{R}$ is monotone. Show that $\int_a^b f(x)dx$ exists.

Problem 2. Suppose that $f(x)$ is continuous on $[a, b]$. Show that $f(x)$ is uniformly continuous on $[a, b]$.

Problem 3. Use Romberg integration with 2^5 intervals to estimate the following integral. How accurate is the estimate?

$$\int_1^2 \sin(x^2)dx$$

Problem 4. Let $f(x)$ be defined as below.

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$$

Show that $f(x)$ is not Riemann integrable on $[0, 1]$.

Problem 5. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable and that $f'(x)$ is bounded on $\mathbb{R}$. Show that $f(x)$ is uniformly continuous on $\mathbb{R}$.