MAA 4212 – KEESLING – TEST 2

Name _____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning even though the final answer may be wrong. Credit will be deducted for incorrect work even though the answer may be right.

Problem 1. State and prove the Contraction Mapping Theorem.

Problem 2. Show that the the following power series converges to the function $\ln(1+x)$.

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

For what values of x does the power series converge? Explain.

Problem 3. Consider the differential equation

$$\frac{dx}{dt} = \sin(t) \cdot \cos(x) \quad x(0) = \pi.$$

Find the polynomial of sixth degree, $a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6$, that best approximates the solution of this differential equation. Give the exact coefficients. [Hint, use the TaylorMethod program downloaded onto your calculator.]

Problem 4. What is the centroid of the curve $x^2 + y^2 = 1$ with $x \geq 0$ and $y \geq 0$? What is the centroid of the area bounded by this curve together with the $x-$axis and $y-$axis?

Problem 5. Determine the volume of a sphere of radius R using Cavalieri's Method.