MAA 4212 QUIZ 1 SPRING 2018

Problem 1. Determine the integral $\int_0^1 f(x)dx$ where $f(x)$ is the **Devil's Staircase Function**.

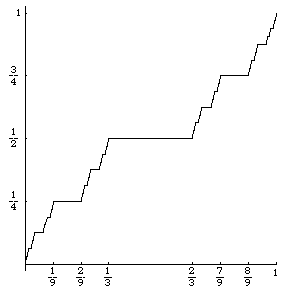

FIGURE 1. The Devil's Staircase

Problem 2. State **Cavalieri's Principle**. Use Cavalieri's Principle to determine the volume of a sphere of radius R.

Problem 3. Use Cavalieri's Principle to determine the volume of a solid torus determined by rotating a $2-$dimensional disk of radius r around the $z-$axis assuming that the distance to the center of the disk from the $z-$axis is R. Also assume that $R - r > 0$.

Problem 4. Determine the area under the curve using upper and lower sums for the function $f(x) = x^5$ over the interval $[0, 1]$.

Problem 5. Prove the following. Suppose that $F(x)$ and $G(x)$ are continuous over $[a, b]$ and that

$$\frac{dF(x)}{dx} \equiv \frac{dG(x)}{dx}$$

for all $x \in (a, b)$. Show that there is a constant C such that $G(x) \equiv F(x) + C$ for all $x \in [a, b]$.

Problem 6. Let $f(x)$ be the **Dirichlet Function** on $[0, 1]$. Show that the Riemann Integral of $f(x)$, $\int_0^1 f(x)dx$, does not exist.