MAA 4212 QUIZ 1 SPRING 2018

Problem 1. Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$. What is the radius of convergence R for $f(x)$? What is the power series for $\ln(1 + x)$ and what is the radius of convergence?

Problem 2. What is the power series for $\arctan(x)$? What is the radius of convergence for this series?

Problem 3. State the **Taylor Remainder Theorem**. Apply the theorem to show that $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ for all x.

Problem 4. Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $g(x) = \sum_{n=0}^{\infty} b_n x^n$. Suppose that each of these series have radius of convergence R_f and R_g, respectively. Does $f(x) \cdot g(x)$ have a power series? If so, what is the radius of convergence and what are the coefficients?

Problem 5. Show that $\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ is valid for all x.

Problem 6. Let $f(x)$ be defined by

$$f(x) = \begin{cases} \exp\left(\frac{-1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Show that there is no power series centered at 0 representing $f(x)$.

Problem 7. Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is valid for $(-R, R)$ with $0 < R < \infty$. Let $x_0 \in (-R, R)$. Show that there is a power series $\sum_{n=0}^{\infty} b_n (x - x_0)^n$ with $f(x) = \sum_{n=0}^{\infty} b_n (x - x_0)^n$ on an open interval about x_0. What is the radius of convergence?