MAA 4212 QUIZ 3 SPRING 2018

Problem 1. Let X be a compact metric space and let $C(X,\mathbb{R})$ be the collection of all continuous functions $f : X \to \mathbb{R}$. For $f, g \in C(X, \mathbb{R}$ define $d(f,g) = \sup\{|f(x) - g(x)| \big| x \in X\}$. Show that $d(f,g)$ is a metric on $C(X,\mathbb{R})$.

Problem 2. Suppose that $\{f_n\}$ converges to f in $C(X,\mathbb{R})$ in the metric $d(f,g)$ in Problem 1. Show that $\{f_n\}$ converges to f uniformly on X.

Problem 3. Let $I = [a,b] \subset \mathbb{R}$. Suppose that $\{f_n\}$ converges to f in $C(I,\mathbb{R})$ in the metric $d(f,g)$. Show that $\int_a^b f_n(x)dx \to \int_a^b f(x)dx$ as $n \to \infty$. Think of $\int_a^b ()dx : C(I,\mathbb{R}) \to \mathbb{R}$ as a function from $C(I,\mathbb{R})$ to $\mathbb{R}$. Is this function continuous?

Problem 4. Show that $C(X,\mathbb{R})$ is a complete metric space with the metric $d(f,g)$.

Problem 5. Let $I = [a,b] \subset \mathbb{R}$. Show that the polynomials on I are dense in $C(I,\mathbb{R})$.