

MAA 4212 – KEESLING – TEST 1

Name _____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning even though the final answer may be wrong. Credit will be deducted for incorrect work even though the answer may be right.

Problem 1. State **Cavalieri's Principle**. Determine the volume of a sphere of radius R using this principle.

Problem 2. Use Pappus' Theorem to determine the surface area of a torus. Assume that the torus is formed by rotating a circle around the z -axis where the circle has radius b and the center of the circle is distance a from the z -axis with $a > b$.

Problem 3. State the **Fundamental Theorem of Calculus**.

Problem 4. Define the **Dirichlet Function** and show that it is not Riemann integrable over $[0, 1]$.

Problem 5. Let $f(x)$ be a bounded function over the interval $[a, b]$. Define when $f(x)$ is **Riemann integrable**. Show that if $f(x)$ is monotone increasing over the interval $[a, b]$, then $f(x)$ is Riemann integrable over $[a, b]$.