MAA 4212 TEST 2 – JAMES KEESLING

NAME _____

Work all problems and show all work. Each problem is worth 20 points. Partial credit will be given for correct reasoning. Credit will be deducted for statements and reasoning that are incorrect.

Problem 1. State the **Taylor Remainder Theorem**. Use the theorem to show that $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ represents the $\sin(x)$ function for all $x \in \mathbb{R}$.

Problem 2. Use **Picard Iteration** with five iterations to approximate a solution to the following differential equation

$$\frac{dx}{dt} = t \cdot x$$
$$x(0) = 2$$

.

Problem 3. State the **Baire Category Theorem**. Use the theorem to show that $\mathbb{R}$ is not countable.

Problem 4. Show that $\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ for all $|x| < 1$.

Problem 5. Give an example of a function that is continuous and everywhere infinitely differentiable that does not have a power series representation around $x_0 = 0$. Explain your answer.