Problem 1. State the Taylor Remainder Theorem. Use the theorem to show that 
\[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \] represents the \( \sin(x) \) function for all \( x \in \mathbb{R} \).

Problem 2. Use Picard Iteration with five iterations to approximate a solution to the following differential equation
\[
\frac{dx}{dt} = t \cdot x
\]
\[ x(0) = 2 \]
Problem 3. State the Baire Category Theorem. Use the theorem to show that $\mathbb{R}$ is not countable.

Problem 4. Show that $\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ for all $|x| < 1$. 
Problem 5. Give an example of a function that is continuous and everywhere infinitely differentiable that does not have a power series representation around $x_0 = 0$. Explain your answer.