Problem 1. Let $h$ be a continuous function $h : \mathbb{R}^n \to \mathbb{R}^n$. Let $x_0 \in \mathbb{R}^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 2. Solve the equation $x^4 = 3$ by the Newton-Raphson method. Give your starting point and the iterates of the Newton function. Give your final solution to ten digits.
Problem 3. Determine the polynomial $p(x)$ of degree 5 passing through the points \{(0,0), (\frac{1}{4}, 0), (1, 0), \left(\frac{3}{2}, 1\right), (2, 0), \left(\frac{5}{2}, 0\right)\}.

Problem 4. Determine the closed Newton-Cotes coefficients for nine points, \{a_0, a_1, \ldots, a_8\}. 
Problem 5. Use the Romberg method for approximating the integral to estimate the integral $\int_{0}^{2} \sin(x^2) \, dx$. Use thirty-two points (i.e., $n = 5$). Indicate the estimate of the integral by the method and how many digits are likely correct. Explain.

Problem 6. Determine the coefficients to compute the second derivative of $f(x) = \sin(x^2)$ at $a = 2$ using the points \{a − 2h, a − h, a, a + h, a + 2h\}. Give the estimate of the second derivative as a function of $h$. Determine the best value of $h$ for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of $h$?
**Problem 7.** Give a numerical solution of the differential equation $\frac{dx}{dt} = f(t,x) = t \cdot x^2$ with $x(0) = 2$. Solve using the Taylor method of order three. Solve on the interval $[0,1]$ using $h = \frac{1}{5}$ and $n = 5$.

**Problem 8.** Determine the steady-state probabilities for an M/M/1/FIFO queue. Assume that the arrival rate is $\alpha$, the service rate is $\sigma$, and that $\sigma > \alpha > 0$. Use the Queue program to simulate a queueing system for M/M/1/FIFO with $\alpha = 9$ and $\sigma = 10$. Use $t = 100$ in the program. How do the results compare with the theoretical calculations for $\{P_n\}_{n=0}^\infty$? Compare the theoretical and experimental results for $p_0, p_1, p_2, p_3$. 

Problem 9. Consider the queueing system with an infinite number of servers M/M/$\infty$. Suppose that the arrival rate is $\alpha$ and the service rate for each server is $\sigma$. Determine the steady-state probabilities $\{p_n\}_{n=0}^{\infty}$ for this system.

Problem 10. In Gambler’s Ruin two players engage in a game of chance in which A wins a dollar from B with probability $p$ and B wins a dollar from A with probability $q = 1-p$. There are $N$ dollars between A and B and A begins the $n$ dollars. They continue to play the game until A or B has won all of the money. What is the probability that A will end up with all the money assuming that $p \neq q$. Assume that $p = .52$ What is the probability that A will win all the money if $n = $15 and $N = $4,000?