MAD 4401 FINAL FALL 2018 - JAMES KEESLING

NAME ______________________________

Do all problems and show all work. Each problem is worth 10 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Two individuals, A and B, play a game of chance. Each time A wins with probability .52 and loses with probability .48. Each time A wins he collects \$1.00 from B. Each time A loses he pays B \$1.00. At the beginning of play A has \$20.00 and B has \$980.00. They play until either A or B has all the money. What is the probability that A will win all the money.

Problem 2. Assume a queueing system with Poisson arrival rate of α, with with an infinite number of servers serving at a rate of σ. This is an M/M/∞ queue. Derive the long-term probability p_n that there are n customers in the system.

Problem 3. A test for a disease is positive with probability .98 when administered to a person with the disease. It is negative with probability .95 when administered to a person not having the disease. Suppose that the disease occurs in 1 out of 1,000 people. Suppose that the test is administered to a person at random and the test is positive. What is the probability that the person has the disease?

Problem 4. Solve the following first-order system of differential equations: $\frac{dx}{dt} = -y$ and $\frac{dy}{dt} = x$ with initial conditions $x(0) = 0$ and $y(0) = 2$. Use the program **picard2** to estimate a solution using 5 iterations. Give the estimates of $x(t)$ and $y(t)$ for the fifth iteration.

Problem 5. Solve the differential equation $\frac{dx}{dt} = f(t, x) = t \cdot \cos(x)$ with $x(0) = 3$.

(a) Solve using the Taylor method of order 4 on the interval $[0, 1]$ with $h = \frac{1}{3}$ and $n = 3$. Give the estimated value of the solution at each value of t to seven digits accuracy.

(b) Determine the Taylor polynomial of degree 8 that approximates the solution.

Problem 6. Determine a solution of the equation $x^4 - 2x^3 + 2x^2 - x = 5$ using the Newton-Raphson method. Give your starting point and circle the final answer.

Problem 7. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 8. Determine the polynomial $p(x)$ of degree 5 passing through the points $\left\{(0,1), \left(\frac{1}{2},0\right), (1,0), \left(\frac{3}{2},0\right), (2,0), \left(\frac{5}{2},0\right)\right\}$.

Problem 9. Estimate $\int_0^1 \cos(x^4)dx$ using Romberg Integration using 2^5 subintervals. Give the first column of the result to 5 digits and the last two columns to 12 digits. Circle the best answer and say how many digits you feel are correct.

Problem 10. Determine the coefficients to compute the first derivative of $f(x) = \sin(x^3)$ at $a = 1$ using the points $\{a-3h, a-h, a, a+h, a+3h\}$. Give the estimate of the derivative as a function of h.