MAD 4401 FINAL - JAMES KEESLING

NAME ______________________________

Do all problems. Each problem is worth 10 points. Partial credit will be given for correct reasoning when the final answer may be incorrect. Credit will be deducted if reasoning is wrong even if the final answer is correct.

Problem 1. Determine the coefficients to compute the second derivative of $f(x) = \cos(x^2)$ at $a = 1$ using the points $\{a-4h, a-h, a, a+h, a+4h\}$. Give the estimate of the derivative as a function of h. Determine the best value of h for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of h?

Problem 2. Assume a queueing system with Poisson arrival rate of α and a single server with an exponential service rate σ. Assume that $\sigma > \alpha > 0$. This is an M/M/1/FIFO queue. Derive the steady-state probabilities for n, $\{\overline{p}_n\}_{n=0}^{\infty}$ for this system.

Problem 3. A test for a disease is positive with probability .93 when administered to a person with the disease. It is positive with probability .04 when administered to a person not having the disease. Suppose that the disease occurs in 10 out of 200,000 people. Suppose that the test is administered to a person at random and the test is positive. What is the probability that the person has the disease?

Problem 4. Determine the natural cubic spline through the points $\{(0,-2),(1,1),(2,2),(3,0)\}$. Give the cubic polynomial for the spline on each of the intervals $\{[0,1],[1,2],[2,3]\}$.

Problem 5. Solve the equation $x^3 + 2 = \cos x$ by the Newton-Raphson method. Give the Newton function. Find a starting point for which the method converges. Give the starting point and the iterations with five digits accuracy. Give the final answer to twelve digits and circle the final answer.

Problem 6. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 7. Give the Legendre polynomial of degree 5. Determine the points and weights of for Gaussian quadrature for 5 points in the interval $[-1, 1]$.

Problem 8. Estimate $\int_0^\pi \cos(x^3)dx$ using Romberg Integration using 2^7 subintervals. Give the first column of the result to 5 digits and the last two columns to 12 digits. Circle the best answer.

Problem 9. Solve the differential equation $\frac{dx}{dt} = f(t,x) = t^3 \cdot x$ with $x(0) = 1$. Solve using Picard iteration for five iterations.

Problem 10. Solve $\frac{dx}{dt} = M \cdot x$ with $M = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and with $x(0) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. Solve on $[0,1]$ with $h = \frac{1}{2}$ and $n = 2$.