MAD 4401 FINAL EXAM

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NAME: ________________________________

Work all problems and show all work. Each problem is worth ten points. Partial credit will be given for correct reasoning. Partial credit will be deducted for incorrect statements and reasoning.

Problem 1. Let h be a continuous function $h : R^n \to R^n$. Let $x_0 \in R^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 2. Solve the equation $5\sin(x^3) = 2$ by the Newton-Raphson method. Give your starting point and the iterates of the Newton function. Give your final solution to ten digits.

Problem 3. Determine the polynomial $p(x)$ of degree 6 passing through the points $\left\{(0,0)\,,\left(\frac{1}{3},0\right),(1,0)\,,\left(\frac{3}{2},1\right),(2,0)\,,\left(\frac{7}{2},0\right),(4,2)\right\}$.

Problem 4. Determine the normalized closed Newton-Cotes coefficients for eight points, $\{a_0, a_1, \ldots, a_7\}$.

Problem 5. Use the Romberg method for approximating the integral to estimate the integral $\int_0^1 \exp(-x^2)dx$. Use sixty-four points (i.e., $n = 6$). Indicate the estimate of the integral by the method and how many digits are likely correct. Explain.

Problem 6. Determine the coefficients to compute the first derivative of $f(x) = \cos(x^2)$ at $a = 3$ using the points $\{a - 2h, a - h, a, a + h, a + 3h\}$. Give the estimate of the first derivative as a function of h. Determine the best value of h for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of h? Assume that the error in computing $\cos(x^2)$ is 10^{-14}.

Problem 7. Give a numerical solution of the differential equation $\frac{dx}{dt} = f(t,x) = \cos(t^2) \cdot x$ with $x(0) = 2$. Solve using the Taylor method of order six. Solve on the interval $[0,1]$ using $h = \frac{1}{3}$ and $n = 3$.

Problem 8. Determine the steady-state probabilities for an M/M/1/FIFO queue. Assume that the arrival rate is α, the service rate is σ, and that $\sigma > \alpha > 0$. Use the **Queue** program to simulate a queueing system for M/M/1/FIFO with $\alpha = 7$ and $\sigma = 10$. Use $t = 100$ in the program. How do the results compare with the theoretical calculations for $\{\overline{p}_n\}_{n=0}^{\infty}$? Compare the theoretical and experimental results for p_0, p_1, p_2, p_3.

Problem 9. Consider the queueing system with an infinite number of servers M/M/∞. Suppose that the arrival rate is α and the service rate for each server is σ. Determine the steady-state probabilities $\{\overline{p}_n\}_{n=0}^{\infty}$ for this system.

Problem 10. In **Gambler's Ruin** two players engage in a game of chance in which A wins a dollar from B with probability p and B wins a dollar from A with probability $q = 1 - p$. There are N dollars between A and B and A begins with n dollars. They continue to play the game until A or B has won all of the money. What is the probability that A will end up with all the money assuming that $p = .52$, A's stake is $n = 10$ and $N = 1,000$? What is the probability that A will win all the money if $n = 25$ and $N = 1,000$?