Problem 1. Let $h$ be a continuous function $h : \mathbb{R}^n \to \mathbb{R}^n$. Let $x_0 \in \mathbb{R}^n$. Suppose that $h^n(x_0) \to z$ as $n \to \infty$. Show that $h(z) = z$.

Problem 2. Solve the equation $5 \sin(x^3) = 2$ by the Newton-Raphson method. Give your starting point and the iterates of the Newton function. Give your final solution to ten digits.
Problem 3. Determine the polynomial $p(x)$ of degree 6 passing through the points $\{(0,0), (\frac{1}{3}, 0), (1, 0), (\frac{2}{3}, 1), (2, 0), (\frac{5}{2}, 0), (4, 2)\}$.

Problem 4. Determine the normalized closed Newton-Cotes coefficients for eight points, $\{a_0, a_1, \ldots, a_7\}$. 
Problem 5. Use the Romberg method for approximating the integral to estimate the integral \( \int_0^1 \exp(-x^2) \, dx \). Use sixty-four points (i.e., \( n = 6 \)). Indicate the estimate of the integral by the method and how many digits are likely correct. Explain.

Problem 6. Determine the coefficients to compute the first derivative of \( f(x) = \cos(x^2) \) at \( a = 3 \) using the points \( \{a - 2h, a - h, a, a + h, a + 3h\} \). Give the estimate of the first derivative as a function of \( h \). Determine the best value of \( h \) for the greatest accuracy of the answer. How many digits accuracy can you expect with this choice of \( h \)? Assume that the error in computing \( \cos(x^2) \) is \( 10^{-14} \).
Problem 7. Give a numerical solution of the differential equation \( \frac{dx}{dt} = f(t, x) = \cos(t^2) \cdot x \) with \( x(0) = 2 \). Solve using the Taylor method of order six. Solve on the interval \([0, 1]\) using \( h = \frac{1}{3} \) and \( n = 3 \).

Problem 8. Determine the steady-state probabilities for an M/M/1/FIFO queue. Assume that the arrival rate is \( \alpha \), the service rate is \( \sigma \), and that \( \sigma > \alpha > 0 \). Use the \texttt{Queue} program to simulate a queueing system for M/M/1/FIFO with \( \alpha = 7 \) and \( \sigma = 10 \). Use \( t = 100 \) in the program. How do the results compare with the theoretical calculations for \( \{p_n\}_{n=0}^{\infty} \)? Compare the theoretical and experimental results for \( p_0, p_1, p_2, p_3 \).
Problem 9. Consider the queueing system with an infinite number of servers M/M/∞. Suppose that the arrival rate is α and the service rate for each server is σ. Determine the steady-state probabilities \( \{p_n\}_{n=0}^{\infty} \) for this system.

Problem 10. In Gambler’s Ruin two players engage in a game of chance in which A wins a dollar from B with probability \( p \) and B wins a dollar from A with probability \( q = 1 - p \). There are \( N \) dollars between A and B and A begins with \( n \) dollars. They continue to play the game until A or B has won all of the money. What is the probability that A will end up with all the money assuming that \( p = .52 \), A’s stake is \( n = 10 \) and \( N = 1,000 \)? What is the probability that A will win all the money if \( n = 25 \) and \( N = 1,000 \)?